

# Numerical study on ultimate buckling strength of stiffened steel plate considering initial deflection and residual stress

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**ABSTRACT:** Ultimate buckling strength of simply supported stiffened steel plate under uniformly distributed axial compressive load was investigated in this paper focusing on the plate like buckling behavior. Initial imperfections due to fabrication process i.e. initial out of plane deflection and residual stress were considered as two independent variables upon which variability of the ultimate buckling strength depends. Magnitudes of initial deflection and residual stress were simulated in the numerical analyses to represent their probabilistic distributions reported in past experiments by Fukumoto et al. and Nara et al. Nonlinear elasto-plastic analysis have been carried out for stiffened plate model having different width-thickness ratio parameter  $R_r$  in the range of 0.4 to 1.4 where each model encompasses combination of initial deflection and residual stress. Geometric as well as material nonlinearity has been considered in the study. Finally, ultimate buckling strength for different  $R_r$  values have been estimated through numerical analysis and compared with the experimental results from previous study.

## 1 INTRODUCTION

Structural steel has been used advantageously than other construction materials for more than last hundred years due to its high strength and ductility. Most of the modern landmark structures in the world are made of steel structure which shows its significance in the current construction industry. Structural parts widely used in steel structures such as bottom flange of steel box girders, orthotropic steel deck of bridges, hull of ships and other marine structures etc. are constructed from stiffened steel plates due to high strength to weight ratio. In the continuous support zone of a bridge, bottom flange of steel box girder is subjected to axial compressive load and design of stiffened bottom flange in such cases is governed by the buckling strength of the stiffened steel plate. During the last 40 years, extensive research has been conducted on the buckling behavior and ultimate load carrying capacity of stiffened steel plates. However, numerous studies concentrated on either experimental results or deterministic finite element analysis result where consideration of the combined effect of initial out of flatness deflection and residual stress were absent which are known as a source of variability of ultimate buckling strength.

Longitudinally stiffened steel plate as shown in Figure 1 having aspect ratio  $\alpha = a/b \leq 1.0$  exhibits plate like buckling behavior (Beg et al. 2010). Plate like buckling of longitudinally stiffened plates means a global buckling of the whole panel composed of a plate and stiffeners (Fig. 2). In plate like buckling, longitudinal edges of the stiffened steel plates are considered as simply supported while in column like buckling the longitudinal edges are considered unsupported. Most of the previous studies i.e. Fukumoto et al. (1977), Nakai et al. (1985) emphasized on column like buckling behavior but it is also important to investigate the plate-like buckling behavior.

Japan's most influential reference for the design of civil engineering structures is the "Japanese Specification for Highway Bridges (JSHB)" according to which the standard ultimate strength curve of a stiffened steel plate governed by local buckling is given by the following equations (JSHB. 2002)

$$\frac{\sigma_{cr}}{\sigma_y} = 1.0 \quad (R_r \leq 0.5) \quad (1.1)$$

$$\frac{\sigma_{cr}}{\sigma_y} = \frac{0.5}{R_r^2} \quad (1.0 < R_r) \quad (1.3)$$

Here  $\sigma_{cr}$  is the ultimate buckling strength,  $R_r$  is the width-thickness ratio parameter given by the following formula

$$R_r = \frac{b}{t} \sqrt{\frac{\sigma_y}{E} + \frac{12(1-\mu^2)}{\pi^2 k_r}} \quad (2)$$

where,  $b$  is the overall width of stiffened plate;  $t$  represents the thickness of stiffened plate;  $\sigma_y$ ,  $E$  and  $\mu$  represents the yield strength, modulus of elasticity and Poisson's ratio of stiffened plate respectively and  $k_r$  is the buckling coefficient given by  $4n^2$  where  $n$  is the number of panels divided by the longitudinal stiffeners.

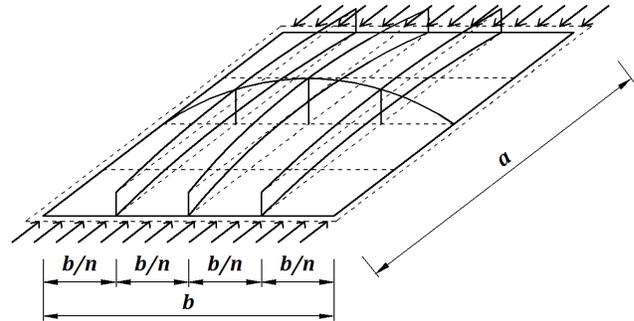
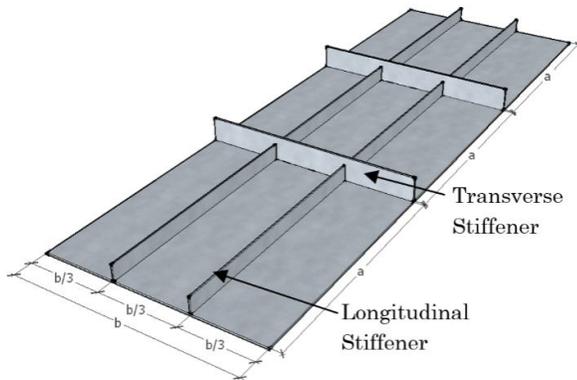


Figure 1. Stiffened steel plate used in the bottom flange of steel box girder      Figure 2. Plate-like buckling of a stiffened plate

Equations 1.1-1.3 was originally proposed by Kanai&Otsuka (1977) based on their experimental results on ultimate load carrying capacity of 43 stiffened steel plates under uniaxial loading having different  $R_r$  values in the range of 0.32 to 1.74. Fukumoto et al. (1977) reported experimental result of longitudinally stiffened steel plates having low  $R_r$  values in the range of 0.46 to 0.78 with relatively rigid stiffeners. It was observed from the experimental results that the current design equation for ultimate buckling strength in JSHB is underestimated for  $R_r$  values  $> 0.8$  and over estimated for lower  $R_r$  values. Nakai et al. (1985) also reported that JSHB doesn't predicts good result while compared with elasto-plastic large deflection analysis results of multi-stiffened plate.

In addition to the limitations stated above, there is one more important reason which necessitates the re-examining of ultimate strength design equation. Now-a-days there is a tendency of using thicker plates in bridge construction. In fact, the regulation for the maximum plate thickness in the JSHB has been increased from 50 to 100 mm since 1996 (JRA. 1996). But the experimental results used to establish the current design equation in JSHB was obtained using thinner plate specimens. Nevertheless, the current design equation has not been revised yet to account for the use of thicker plates.

In order to investigate the ultimate buckling strength, two methods can be applied i.e. experimental and numerical. To obtain reliable experimental result, it is necessary to conduct sufficiently large number of tests. Besides, it is also very important to measure all the parameters that affects the ultimate buckling strength such as width-thickness ratio parameter, initial out of plane deflection, residual stress etc. for every tests. Even though it possible to measure and control the geometric properties of the plate, measurement of residual stress is particularly costly due to requirement of special equipment and extremely scrupulous techniques with high accuracy. Moreover, it is very difficult to control the residual stress level as it depends on many factors such as temperature of welding, thickness of steel plate, ambient condition while manufacturing steel, yielding value of steel material etc. On the other hand, it is convenient to control the parameters affecting ultimate buckling strength in numerical analyses.

Based on the above identified issues, objective of this paper is to investigate the ultimate buckling strength of longitudinally stiffened steel plates with simply supported boundary condition along the longitudinal edge to ascertain plate-like buckling considering the combined effect of initial out of plane deflection and residual stress. Steel plate thickness up to 90 mm will be considered to incorporate the effect of thicker plates. Nonlinear elasto-plastic finite element analysis considering material and geometric nonlinearity will be performed to obtain accurate result.

## 2 STATISTICAL DISTRIBUTION OF INITIAL DEFLECTION AND RESIDUAL STRESS

### 2.1 Initial Out of Plane Deflection

Initial out of plane deflection in the stiffened plate usually occurs during the fabrication process specially after welding. Nara & Komatsu (1988) reported statistical information of initial deflection obtained from measurements of stiffened plate members in steel box girder bridges fabricated during the eighties of last century in Japan. Figure 3 represents the frequency distribution of initial deflection  $\delta_{0l}$  normalized with respect to length of the stiffened plate  $a$  while the deflection mode has waveform of sine half-wave as shown in Figure 2. Two different Weibull distributions were fitted for negative and positive out of plane deflection having 274 and 230 number of observations respectively. Here positive direction means toward the stiffener and vice-versa.

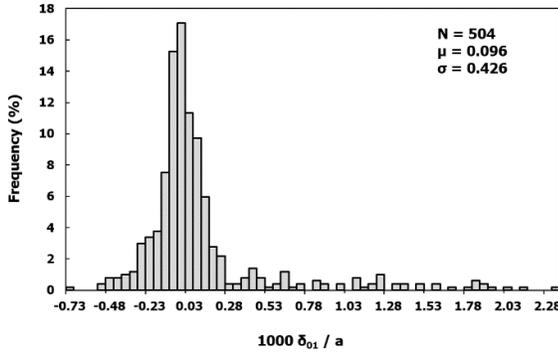


Figure 3. Frequency distribution of initial out of plane deflection

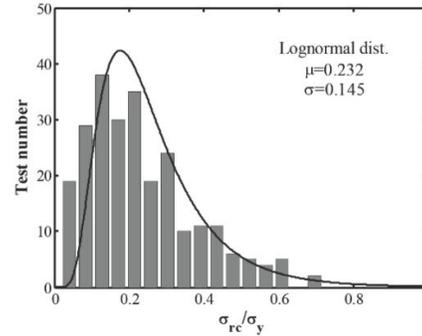


Figure 4. Lognormal distribution of residual stress

### 2.2 Residual Stress

Residual stresses develops predominantly due to the welding process in stiffened steel plate. Primary reason for development of these residual stresses is the differences in the amount the weld metal shrinks as it hardens and cools to the surrounding temperature (Kenno et al. 2010). Actually, statistical information on measurement of residual stresses in steel plate members are very rare. Among published information, Fukumoto & Itoh (1984) shows the greatest number of residual stress measurement data for unstiffened plates of single plates and square box loaded in uniaxial compression. Histogram of residual compressive stress  $\sigma_{rc}$  normalized with respect to the yield stress  $\sigma_y$  has been reported for 248 number of observations. To avoid the negative value of compressive residual stress, a lognormal distribution was produced by Duc et al. (2013) as shown in Figure 4. Mean value and standard deviation of initial out of plane deflection and residual stress are summarized in Table 1 which values were used in the current study for simulating the imperfections.

Table 1. Statistical information of initial imperfections

Imperfections	Mean ( $\mu$ )	Standard deviation ( $\sigma$ )
$1000 \times \delta_{0l} / a$	0.096	0.426
$\sigma_{rc} / \sigma_y$	0.23	0.145

## 3 FINITE ELEMENT ANALYSIS

### 3.1 Model Geometry and Boundary Conditions

Stiffened steel plate of aspect ratio 1.0 with two equidistant flat type longitudinal stiffeners has been chosen to produce plate-like buckling. Due to symmetric geometric and loading condition, instead of modelling whole stiffened panel, the shaded rectangular area as shown in Figure 5 has been modelled for finite element (FE) analysis to reduce the computational time. Width of this rectangular area is equal to half-width of the stiffened plate and length is equal to the length of stiffened plate having a transverse stiffener at middle.

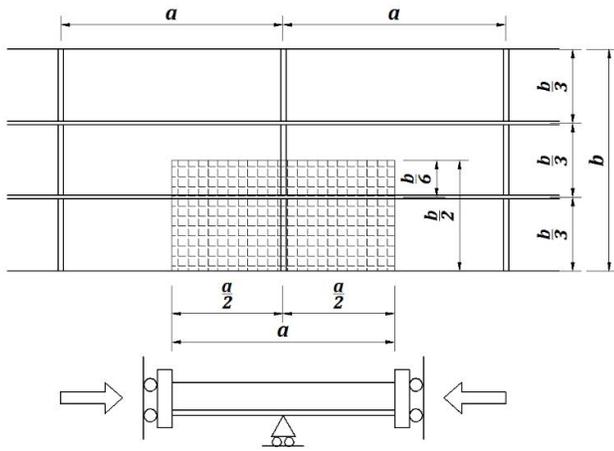


Figure 5. Model Geometry

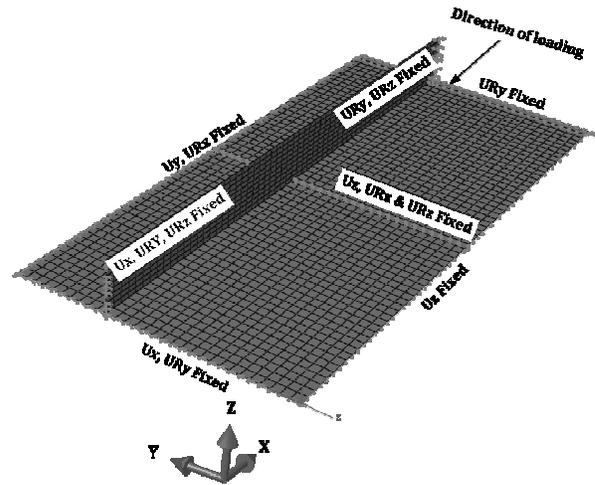


Figure 6. Boundary conditions

Boundary conditions of the model is presented in Figure 6 where  $U_x, U_y, U_z$  denotes translational and  $UR_x, UR_y, UR_z$  denotes rotational degree of freedom along  $X, Y$  and  $Z$  axis. Simply supported boundary condition was applied along the longitudinal outer edge of the stiffened plate. Instead of modelling transverse stiffener, appropriate boundary conditions were applied at the nodal points considering a rigid transverse stiffener.

### 3.2 Material Properties

Steel grade widely used in regular construction i.e. SM490Y as per JSHB code has been selected for analysis. Von mises plasticity and isotropic strain hardening theory was applied to model material nonlinearity. Hardening behavior has been identified from the idealized uniaxial stress ( $\sigma$ )-strain ( $\epsilon$ ) relationships shown in Figure 7 based on test data. In the FE analyses, 4-node isoparametric shell element S4R in ABAQUS has been used to simulate the stiffened plate model.

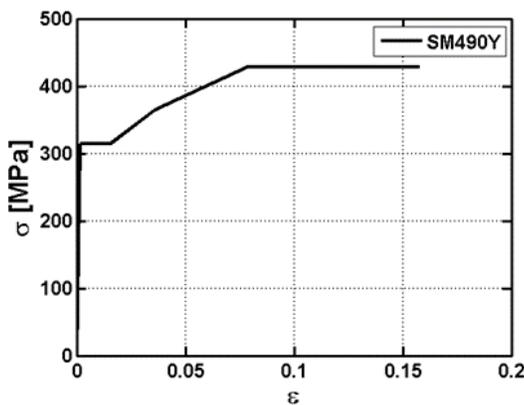


Figure 7. Idealized stress-strain relationship of steel grade SM490Y

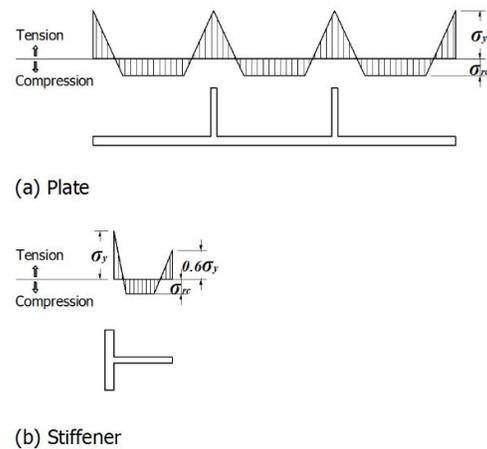


Figure 8. Distribution of residual stress at plate and stiffener

### 3.3 Elastic Buckling Analysis

Before conducting nonlinear elasto-plastic analyses, eigenvalue buckling analyses have been performed to identify the probable buckling mode as well as obtaining the elastic buckling strength. In-plane uniform compressive load was applied through forced displacement. After analyses, buckling modes were observed carefully and it was found that 1<sup>st</sup> buckling mode always shows global whole plate buckling which is the critical case and provides lowest elastic buckling strength  $\sigma_e$ . Deformations for the first two buckling mode and normalized elastic buckling strength ( $\sigma_e/\sigma_y$ ) of a model with  $R_f$  value 0.8 and plate thickness 30mm is presented below as an example.

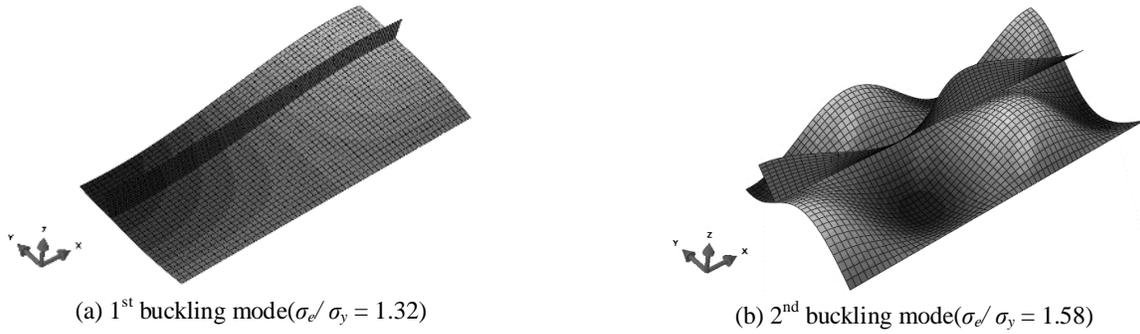


Figure 9. Eigenvalue buckling modes for a model with  $R_r$  value 0.8 and plate thickness 30mm

### 3.4 Simulation of Initial Imperfections

Linear superposition of buckling eigenmodes were used to simulate initial out of plane deflection. As the lowest buckling mode provides the most critical imperfections, so the 1<sup>st</sup> buckling mode is scaled and added to the perfect geometry to simulate initial out of plane deflection. Thus the imperfection has the following form below

$$\Delta x_i = \sum_{i=1}^M w_i \varphi_i \quad (3)$$

where  $\varphi_i$  is the  $i^{\text{th}}$  mode shape and  $w_i$  is the associated scale factor. The value of  $w_i$  is obtained by trials to get the desired initial out of plane deflection as per statistical information given in Table 1.

Residual stresses are considered to be distributed along the width of the stiffened plate and stiffener. Distribution for the longitudinal stress component is assumed to be a triangular tensile zone with a peak tensile stress at the weld balanced by a compressive stress zone between the stiffeners as shown in Figure 8. Based on such theoretical distribution residual stresses were simulated in every element of stiffened plate and stiffener. Magnitude of compressive residual stress was taken from statistical information given in Table 1.

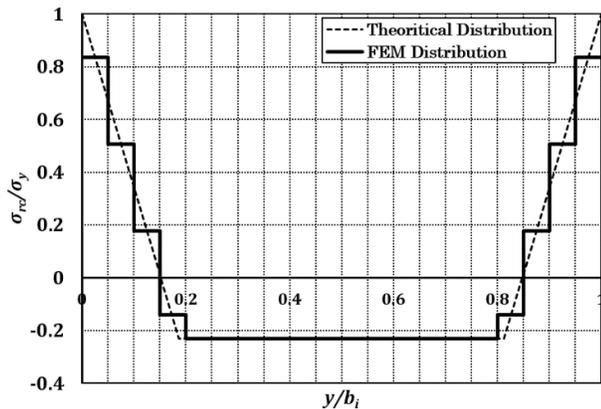


Figure 10. Distribution of residual stress at one subpanel

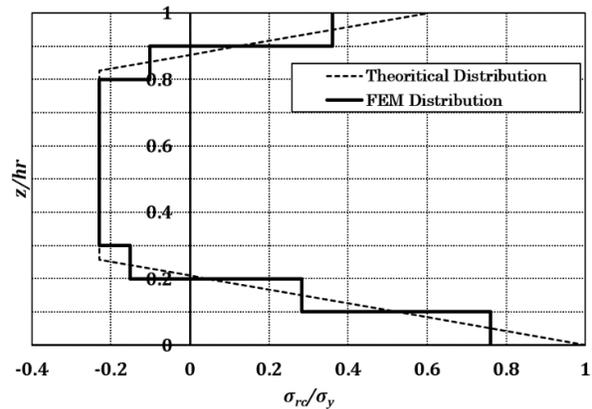


Figure 11. Distribution of residual stress at stiffener

Distribution of residual stresses for mean value case ( $\sigma_{rc}/\sigma_y = 0.23$ ) in one subpanel (area between two longitudinal stiffeners) and in one individual stiffener is shown in Figures 10 and 11 respectively as an example. Here,  $y$  represents the distance along  $Y$  direction starting from either longitudinal support edge or longitudinal stiffeners and  $b_i$  is the width of individual subpanel. Similarly,  $z$  is the distance along  $Z$  direction and  $hr$  is the height of a longitudinal stiffener. Divisions along horizontal and vertical axis in Figure 11 and 12 symbolizes individual elements in subpanel and stiffener. Compressive residual stress is shown in negative sign and vice-versa. At the free edge of longitudinal stiffener, tensile residual stress was taken up to 60% of the yield stress.

### 3.5 Nonlinear Elasto-Plastic Analysis

Nonlinear elasto-plastic FE analyses have been carried out for 23 different stiffened plate models having  $R_r$  value 0.4 to 1.4 with an increment of 0.2. Thickness of stiffened plate varied from 10 mm up to 90 mm for individual  $R_r$  value. Geometric parameters of stiffened plate models were selected from actual bridge data that have been constructed in Japan. In such selection, relative stiffness of longitudinal stiffener ( $\gamma_l$ ) was considered equal to the required relative stiffness of longitudinal stiffener ( $\gamma_{l,req}$ ) given by JSHB code. Uniformly distributed in-plane compressive load was applied through forced displacement like elastic buckling analysis. For each individual model, a set of 12 combinations of initial deflection and residual stress has been chosen. Thus, a total number of  $23 \times 12 = 276$  analyses have been carried out. Combination points are marked by black dots in the Figure 12 where  $\mu$  is the mean value and  $\sigma$  is the standard deviation of respective imperfections.

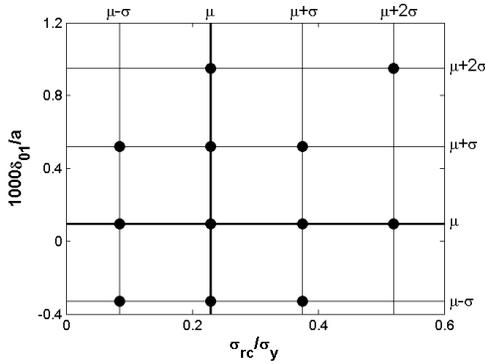
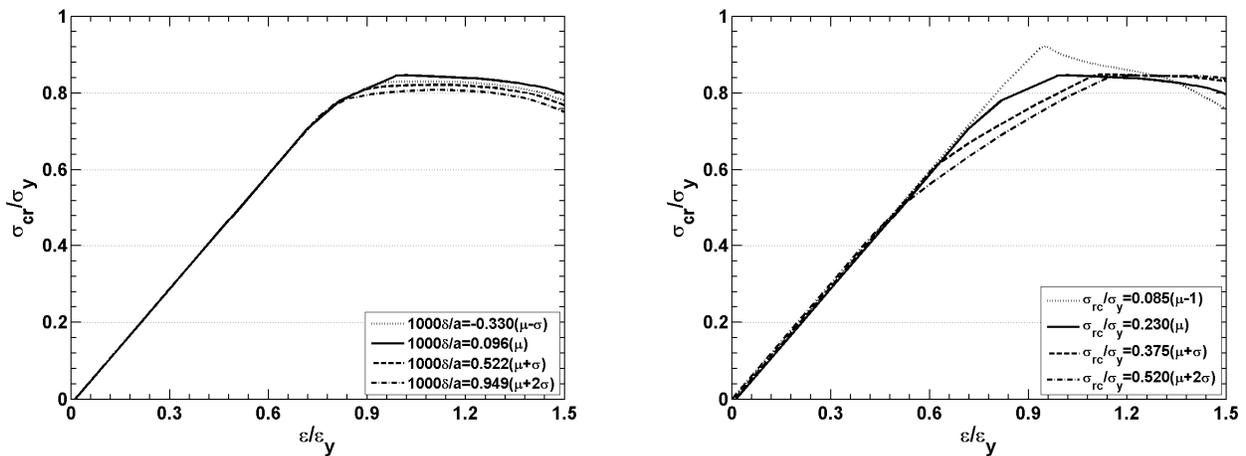


Figure 12. Combination points of residual stress and initial deflection.

## 4 RESULTS AND DISCUSSION

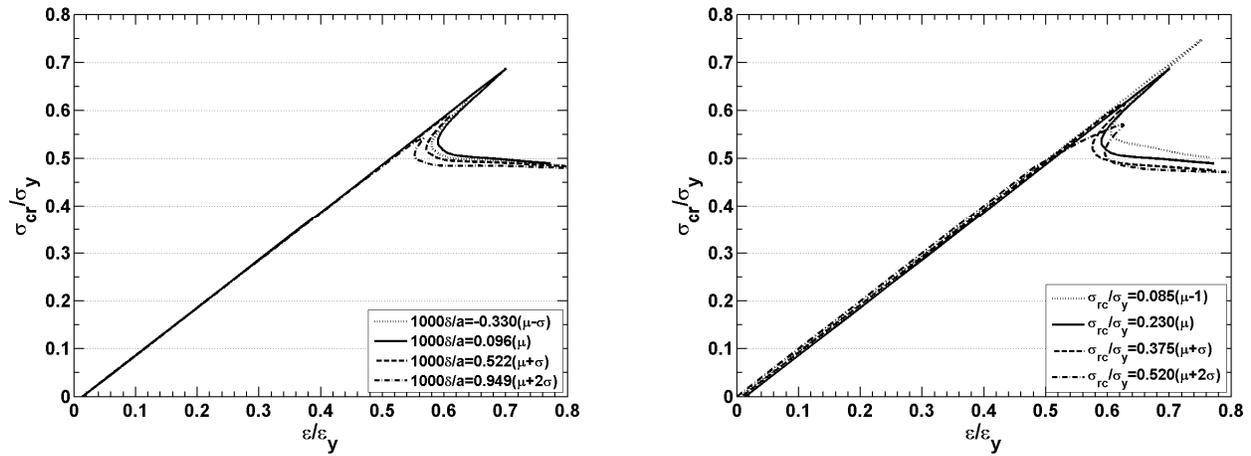
Result of nonlinear elasto-plastic analyses have been represented through normalized stress ( $\sigma_{cr}/\sigma_y$ )–strain ( $\varepsilon/\varepsilon_y$ ) curve where  $\sigma_{cr}$ ,  $\sigma_y$  represents the buckling stress and yield stress and  $\varepsilon$ ,  $\varepsilon_y$  denotes applied axial strain and yield strain of the stiffened plate. Ultimate buckling strength was determined from the peak stress value of such curves. Figure 13 and 14 shows normalized stress-strain curves for 10 mm thick stiffened plate having width-thickness ratio parameter  $R_r = 0.6$  and 0.8 respectively as instance. Figure 13(a) and 14(a) illustrates the effect of variation of initial out of plane deflection on ultimate buckling strength at mean residual stress case. On the other hand, effect of variation of residual stress on ultimate buckling strength at mean initial deflection have been represented in Figure 13(b) and 14(b). Here, mean ( $\mu$ ) and standard deviation ( $\sigma$ ) of initial deflection and residual stress refers to the values presented in Table-1.



(a) Variation of initial deflection at mean residual stress

(b) Variation of residual stress at mean initial deflection

Figure 13. Normalized stress-strain curve for width-thickness ratio parameter  $R_r$  0.6.



(a) Variation of initial deflection at mean residual stress

(b) Variation of residual stress at mean initial deflection

Figure 14. Normalized stress-strain curve for width-thickness ratio parameter  $R_r$  1.0.

It was perceived that the effect of variation of residual stress is more sensitive than the effect of variation of initial out of plane deflection on ultimate buckling strength. The higher the magnitude of imperfection is, either out of plane deflection or residual stress, the lower the critical stress obtained. Moreover, in the post buckling zone for  $R_r$  value up to 0.6, stiffened plate model embodied stable post buckling behavior but for  $R_r$  value equal or greater than 0.8, unstable snap-through behavior was observed.

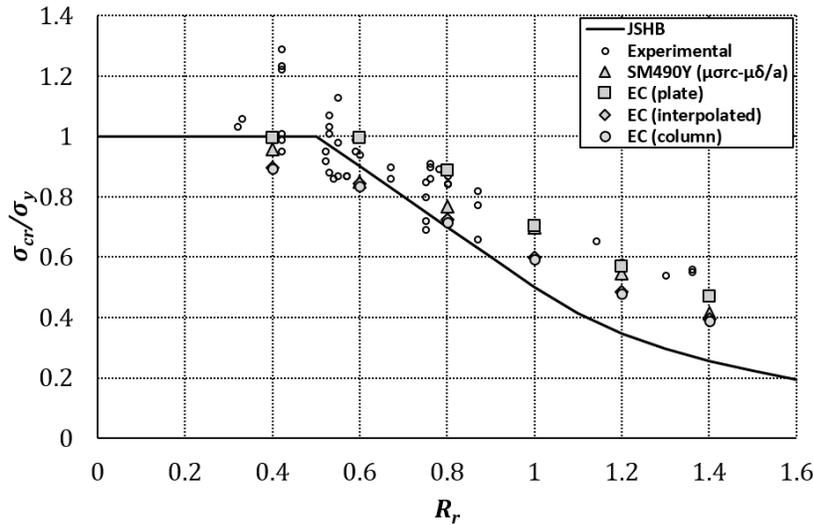


Figure 15. Comparison of FE analysis result with experimental result, JSHB and Eurocode.

Normalized ultimate buckling strength of stiffened steel plate having mean value of imperfections (mean initial deflection and mean residual stress value as described in Table-1) have been plotted with respect to different  $R_r$  value in figure 15 and compared with experimental results by Kanai et al., current provision of JSHB and Eurocode (EC). Eurocode provides different critical buckling strength for plate like and column like buckling and finally interpolate between plate like and column like buckling. It was observed that current numerical analysis result shows good agreement with Eurocode plate like buckling result except  $R_r$  value 0.6 and 0.8. However, all the results i.e. experimental result, numerical result and result obtained from provisions of Eurocode shows that the ultimate buckling strength is underestimated in JSHB code for  $R_r$  value greater than 0.8 and the margin becomes higher as  $R_r$  value increases.

## 5 CONCLUSIONS

Findings of current study as discussed in the preceding section is expected to provide guideline for determining ultimate buckling strength of longitudinally stiffened steel plate having plate like buckling behavior. Dependency of ultimate buckling strength on the variation of initial imperfections have been discussed which has been considered as one of the prime reason behind the discrepancy between experimental result and theoretical result. Now a days, popular trend is to use partial factor format type design equation based on reliability data for structural design. Result of current study can be utilized to determine partial safety factor for ultimate buckling strength after reliability analysis through Monte Carlo simulation which would be the next step of the current ongoing study.

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