IABSE-JSCE Joint Conference on Advances in Bridge Engineering-III, August 21-22, 2015, Dhaka, Bangladesh. ISBN: 978-984-33-9313-5
Amin, Okui, Bhuiyan, Ueda (eds.) www.iabse-bd.org

A new linear redundancy analysis method for steel truss bridges accounting distributed plastic effect

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ABSTRACT: Structural redundancy analysis for old bridges has become crucial since the collapse accident of I-35W Bridge in the United State. A linear redundancy analysis is widely used to assess the redundancy level of bridges. This method assesses the robustness of the damaged bridge by checking the strength of every single member through linear equations of member strength without the effect of cross sections. It is known that the member strength is a nonlinear equation which depends on the shape of the cross section. This study proposed more accurate strength equations for linear redundancy analysis method. The proposed equations accounted for some typical sections, such as I-shape, H-shape, and Box-shape sections. A nonlinear redundancy analysis was also conducted to validate the proposed method. The proposed method results are consistent with results of nonlinear redundancy analysis that is well known as highly accurate approach.

1 INTRODUCTION

Structural redundancy is defined as the ability of a structural system to continue to carry loads after the failure of one or several structural components (Ghosn & Yang, 2014). Bridge redundancy analysis qualifies the structural redundancy level through bridge safety checking after a break occurs on one of its members. A bridge is classified into a redundant bridge or a non-redundant bridge by its redundancy level. In addition, the Fracture Critical Members (FCMs), whose failure may lead to the collapse of the bridge, should be identified after redundancy analysis.

The URS Corporation (2006) and Nagatani, et al. (2008) proposed a linear redundancy analysis method. Robustness of the damaged structure was evaluated by strength checking of the members through two linear equations; one for tensile members, and another for compressive members. Although, the linear method advances on the simplifications, its accuracy is limited because of inaccurate member strength expression. The strength should be nonlinear curves rather than linear lines. In this paper, more accurate plastic strength of members was developed. The proposed strength equations calculate for some typical cross-sectional shapes, such as I-shape, H-shape, and Box-shape. The proposed plastic strength of the members is closer to the real value than the conventional solutions.

2 CONVENTIONAL LINEAR REDUNDANCY ANALYSIS

Conventional linear redundancy analysis is proposed by the URS Corporation (2006) and Nagatani, et al. (2008). The method first analyzes the damaged bridge by a linear elastic analysis. The robustness of the damaged bridge is demonstrated by a redundancy index R. The redundancy index, R, is defined by checking the strength of members. The member strength is expressed with two linear equations; one for tensile members as in Equation (1), and another for compressive members as in Equation (2). If any member gives $R \geq 1$, that member fails in terms of strength. The whole bridge collapses consequently.

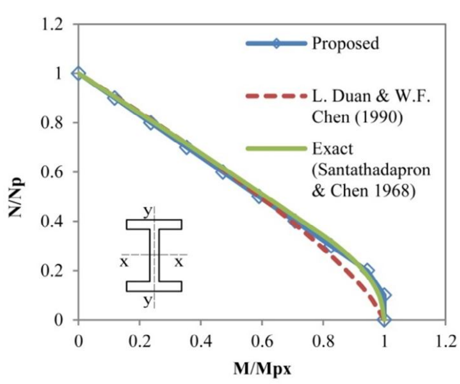
$$\mathrm{R} = \frac{\mathrm{N}}{N_p} + \left(\frac{\mathrm{M}}{M_p}\right)_{\mathrm{out}} + \left(\frac{\mathrm{M}}{M_p}\right)_{\mathrm{in}} \quad (1)$$

$$R = \frac{P}{P_u} + \left(\frac{1}{1 - \frac{P}{P_e}} \right) \left(\frac{M_{eq}}{M_p} \right)_{out} + \left(\frac{1}{1 - \frac{P}{P_e}} \right) \left(\frac{M_{eq}}{M_p} \right)_{in} \quad (2)$$

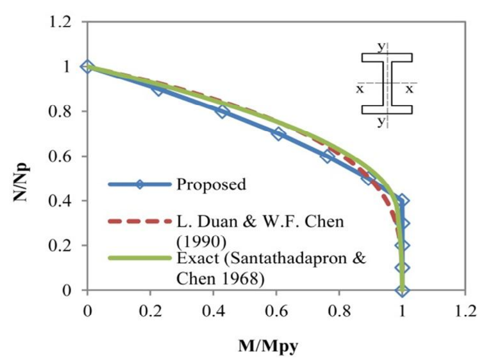
In the above equations, N and P are tensional and compressive forces, respectively, and $(M)_{in}$ and $(M)_{out}$ are the in-plane and out-plane bending moments, respectively. N_p and M_p are the in-plane and out-plane plastic axial strength and full plastic moment strength, respectively. P_e is the Euler buckling load. P_u is the ultimate compressive strength in consideration of the global buckling. The calculation of these capacities can be carried by a standard procedure. The calculation of P_u varies according to the calculation of critical stress. In this study, P_u was computed by the procedure in Japan Road Association (2002). The factor $1/(1-P/P_e)$ is the amplification factor accounting for the second order moment effect that captures the $P-\delta$ effect and the $P-\Delta$ effect. In addition, in the case of steel truss bridges, members rigidly connect at truss joints that prevent lateral moments; thus moments are not uniform along member length and are largest at member ends. This reality must be converted to the equivalent uniform moment, M_{eq}, as in the theoretical case of uniform moments, which is used to determine the strength of members. A reduction factor, C_m, can address this. If moments at member ends are M_1 and M_2, $(|M_1| \geq |M_2|)$, the equivalent moment is $M_{eq} = C_m \times |M_1|$. Such formula by Austin (1961) is widely accepted for calculating reduction factor.

3 PROPOSED METHOD

Many researchers investigated the plastic strength of members. Santathadaporn & Chen (1970) used mathematics to calculate the plastic axial strength and full plastic moments for a rectangular section, and an I-shape section. Chen & Atsuta (1972) used mathematics to draw full yield surfaces for double web and wide flange sections. These approaches are exact solutions. However, they are very complicated, especially in a 3D frame with a combination of the axial force and biaxial moments. Duan & Chen (1990) and George & Tseng (1983) made an approximation for full yield surface equations for most of the typical steel sections. In addition, Sohal, Duan, & Chen (1989) and Duan & Chen (1989) developed the design interaction equations for typical steel sections in the case of compression. The comparisons for I-shape cross sections among above researchers are plotted in Fig. 1. The other section shapes had the same results in comparison. The approximate solution by George & Tseng (1983) is consistent with that calculated from the exact mathematical approach. Hence, this study proposed using the formula by George & Tseng (1983) to check the strength of members in the linear redundancy method. Assuming the strong and weak axes of the cross section are, respectively, the x-axis and y-axis, the strength of the section in uniaxial bending are computed as the following



(a)



(b)

Figure 1. Plastic strength of I-shaped section (a) along strong axis and (b) along weak axis

For the tensile I-shape section

$$\frac{M_{pcx}}{M_{px}} = \min\left(1.18M_{px}\left(1-\frac{N}{N_p}\right),1\right) \tag{3}$$

$$\frac{M_{pcx}}{M_{py}} = \min\left(1.19M_{py}\left(1-\left(\frac{N}{N_p}\right)^2\right),1\right) \tag{4}$$

For tensile Box shape section

$$\frac{M_{pcx}}{M_{px}} = \min\left(1.20M_{px}\left(1-\frac{N}{N_p}\right),1\right) \tag{5}$$

$$\frac{M_{pcy}}{M_{py}} = \min\left(1.20M_{py}\left(1-\frac{N}{N_p}\right),1\right) \tag{6}$$

For compressive members in all types of cross section shapes

$$M_{pux} = M_{px}\left(1-\frac{P}{P_u}\right)\left(1-\frac{P}{P_{ex}}\right) \tag{7}$$

$$M_{puy} = M_{py}\left(1-\frac{P}{P_u}\right)\left(1-\frac{P}{P_{ey}}\right) \tag{8}$$

In the above equations, the full plastic moment strengths of tensile members considering the effect of the tensile axial force, M_{pcx} and M_{pcy}, are computed by Equations (3), (4) (5), and (6). M_{pux} and M_{puy} are moment strengths of compressive members accounting for the effect of the compressive axial force. Most studies have shown that the interaction curve of bending strength and compressive strength is straight rather than such nonlinear curves in cases of tensile members. Hence, in this study, moment strengths of compressive members, M_{pux} and M_{puy}, neglecting the local buckling effect, are defined in Equations (7), (8), in which P_{ux} and P_{uy} were computed in the same procedure as the conventional linear method.

In steel truss bridges, when one member is broken, the moments in the remaining members are increased widely in both the in-plane and out-plane directions. Hence, steel truss members show biaxial bending rather than uniaxial bending moments. In this paper, the strength of members in biaxial bending is assumed to equal the summation of power functions of strength during uniaxial bending moments. For this reason, the redundancy index is proposed as the following.

For tensile members:

$$\mathrm{R} = \left(\frac{\mathrm{M_x}}{M_{pcx}}\right)^{\alpha_x} + \left(\frac{\mathrm{M_y}}{M_{pcy}}\right)^{\alpha_y} \tag{9}$$

For compressive members:

$$\mathrm{R} = \left(\frac{\mathrm{C_{mx}M_x}}{M_{pux}}\right)^{\alpha_x} + \left(\frac{\mathrm{C_{my}M_y}}{M_{puy}}\right)^{\alpha_y} \tag{10}$$

If any member fails its strength, $R \geq 1$, the whole bridge is considered a collapse. M_x and M_y are bending moments in the members around the strong and weak axes. C_{mx}, C_{my} are the reduction factors in x- axis and y-axis respectively. α_x and α_y are the factors addressing the contribution of member strength in the strong axis and weak axis, respectively, to global strength. These factors are proposed by Duan & Chen (1990) as the following.

For the I-shaped section

$$\alpha_x = 1.2 + 2\frac{N}{N_p}; \alpha_y = 2 \tag{21}$$

For the Box shaped section

$$\alpha_x = \alpha_y = 1.7 + 1.5\frac{N}{N_p} \tag{22}$$

4 NUMERICAL VERIFICATION

4.1 *Analytical Model*

4.1.1 *Bridge geometry and finite element model*

A steel truss through bridge built in 1973 in Niigata, Japan, was employed to validate the proposed method. This bridge is a one-span, simple Warren truss bridge. The length of this bridge is 90.2 m, and its width is 9 m. The road clearance is 6.5 m, with two designed travel lanes. The thickness of the concrete deck is 200 mm.

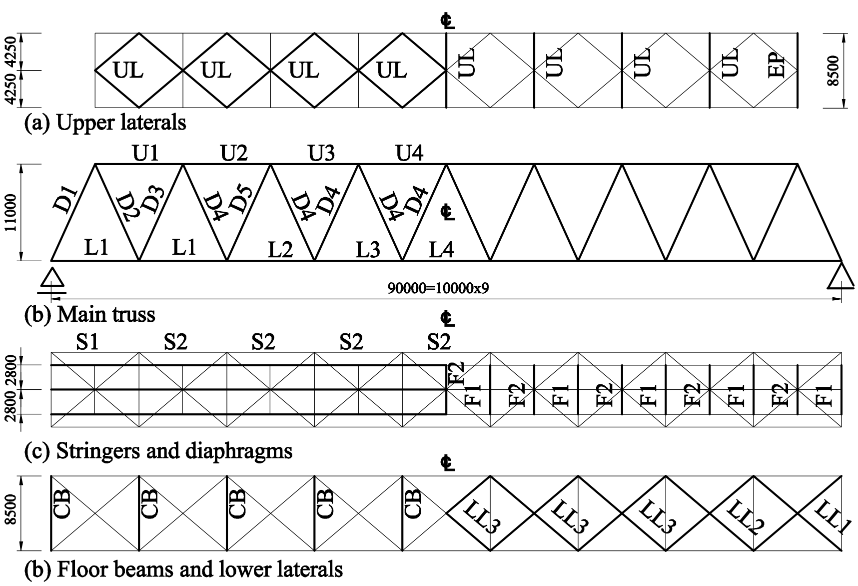


Figure 2. Geometry of the analyzed bridge

The bridge was modeled in the three-dimensional (3D) finite element model, as shown in Figure 3. All structural steel members, such as truss members, laterals, and the deck framing system were in 3D curve beam elements with three nodes in each beam and six degrees of freedom at each node along their center lines. To capture the spread of plasticity over the depth of the beams, layered integration was applied along the beam cross section as well as along shell thickness. Members are rigidly connected at truss joints. The concrete deck is expressed by curved shell elements. Composite action between the reinforced concrete decks to the stringer was addressed by using rigid beams. The same principle was applied to the modeled eccentric connection between the floor beams to the truss joints. All members were meshed in the division size of 125 mm. Hence, by this division size, each chord frame had 80 divisions or 40 elements, and each diagonal had 96 divisions or 48 elements.

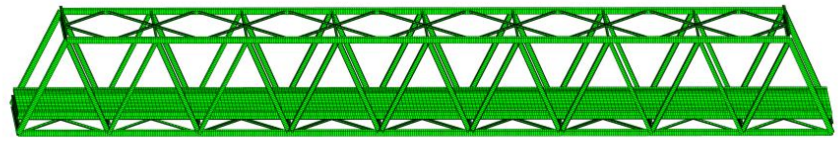


Figure 3. 3D finite element model

4.1.2 *Case studies*

In reality, a virtual break probably appears to any members. The number of members in a truss bridge is large. Analyzing all cases of every single member break one by one could take considerable effort. Hence, selecting suitable virtual break candidates would increase efficiency. This task requires expert experience even that it needs some beforehand analysis. In reality, if suitable candidates cannot be defined, then analysis of every single member will occur. However, this study aimed to investigate the more accurate linear redundancy analysis method, which is not a single redundancy analysis of a particular bridge. For this reason, a few members were assumed to break virtually, as shown in Figure . Six members represent most types of members in the analyzed bridge, except for the lower chords. For a truss through steel bridge, due to very stiff deck system, the loss of the lower chord is supposed to have a less harmful effect than the loss of the other members. That is the reason lower chords were not studied in this research.

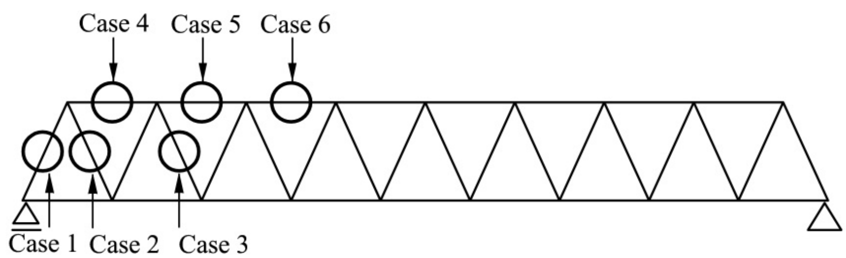


Figure 4. Cases of study

4.1.3 *Materials*

The steel structural members made of steel JIS-SM490A with a distributed mass of $\rho = 7{,}850\ \ (\text{kg/m}^3)$ and a Young's modulus of $E = 2.0 \times 10^{10}\ \ (\text{kg/m}^2)$. The reinforced concrete has a distributed mass of $\rho = 2{,}500\ \ (\text{kg/m}^3)$ and a Young's modulus of $E = 2.3 \times 10^9\ \ (\text{kg/m}^2)$. For nonlinear redundancy, the steel material and reinforced concrete are both assumed as a perfected plastic material because of its simplicity. The structural steel has yield stress of $\sigma_y = 315\ \ \text{MPa}$ while reinforced concrete yields at stress of $\sigma_y = 21\ \ \text{MPa}$.

4.1.4 *Loadings*

Weight of members, live load, and dynamic allowance were examined. The loadings were modeled following Japan Road Association (2002) because the bridge was designed following Japan's specifications. The location of the live load was determined such that the axial force in the target member, which would be assumed as damaged, becomes the maximum. This task can be performed using an influence line of the axial force of the target member. Six live load alignment patterns along bridge length were assigned into the model corresponding to six cases of study. In addition, a so-called release force was employed to address the virtual member break simulation. Loadings were introduced into the model through Phased Analysis. Structural steel members first become active in Phase 1. In this phase, the model subjected the weight of all members, including the concrete deck self-weight. Live load, in addition to concrete deck elements, was added to become active in Phase 2 to perform the structural stage before a virtual break occurrence. This analysis procedure addresses the structural configuration from an initial construction state to the condition before virtual damage. The release force was introduced into the model in Phase 3 to address the virtual break simulation. In this Phased Analysis, the deformation in the former phase is introduced to a phase later than the initial deformation.

4.1.5 *Simulation of member break*

The member failure can appear at any level of live load. This paper assumed the break appeared at a design dead load and factored into a live load multiplied by a dynamic allowance (DAL) of 0.143. The static for the truss member sudden failure is illustrated in Figure . Condition A is the damaged structure. The physical function of the virtual break member was replaced by its sectional force at the member ends in Condition B. The so-called release force, P, that comes from sectional force of virtual break member was applied in the opposite directions in Condition C. Condition A was the superposition of Condition B and C in the simulation of the member break. A load factor α of the release force was increased to eliminate the physical functions of the virtual break. At a load factor of $\alpha = 1.0$, all sectional forces of the virtual member were completely eliminated. This means that the member was physically released. If the virtual break member is a tensile member, the dynamic effect of sudden failure of the member occurs. Increasing the release load factor until a value of $\alpha = 1.854$ should account for this effect. This approach is the most common way proposed by the (URS Corporation, (2006).

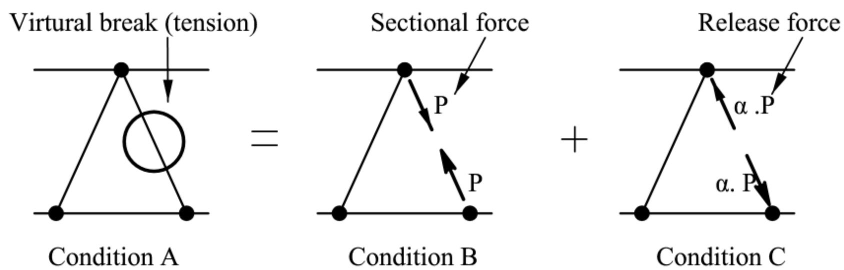


Figure 5. Simulation of virtual break

4.2 *Numerical Analysis*

4.2.1 *Linear redundancy analysis*

Linear redundancy analysis was conducted. The damaged bridge was first analyzed by a linear elastic analysis. The phase option was included in the analysis to address phases of loadings. The redundancy index was computed by both conventional equations and proposed equations. This study purposed to study methods of redundancy analysis, and did not aim to perform a single redundancy analysis. Hence, the study focused on finding the collapse load rather than evaluating the redundancy level of the bridge. This means that the collapse release load factor, in which the first member yielded R=1 during the process of releasing the virtual break member, is the considered parameter.

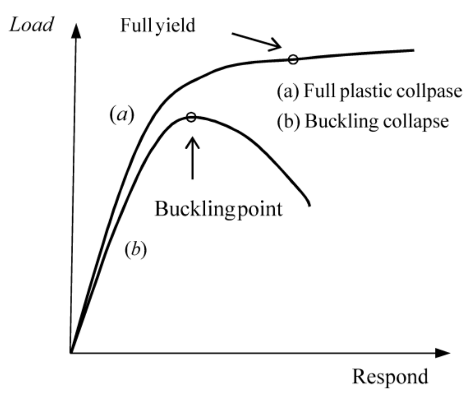


Figure 6. Collapse definition in nonlinear redundancy

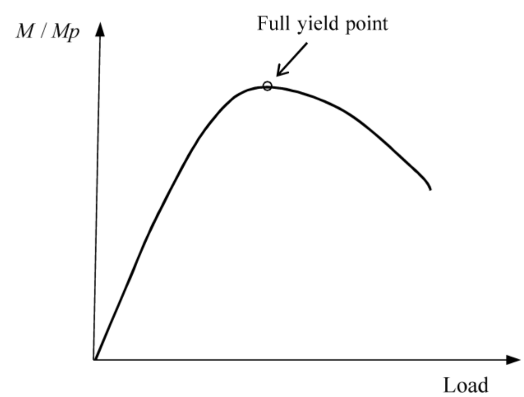


Figure 7. Full yield criteria

4.2.2 *Nonlinear redundancy analysis*

To validate the proposed method, a nonlinear redundancy analysis was conducted. A second- order analysis with phased option analysis was employed to capture such geometric nonlinearity as the $P-\Delta$ effect and $P-\delta$ effect. The initial imperfection of the truss members was accounted by the equation of $L/1000 \times Sin \pi x/L$, in which L is the length of the member. The initial imperfection was introduced in the direction so that it increases the stress in the target member. The member strength in the above linear redundancy method is formed when the section is full-deep yielded or buckling has occurred. Hence, to be comparable, the nonlinear method searched for the point in which the first member has fully plastic yield or buckling, rather than continuous analysis until the final collapse of a standard nonlinear analysis. Typical collapse criteria for nonlinear redundancy are defined in Figure. The structures collapse either due to buckling or plastic collapse. If buckling occurs, the response of the bridge, including strain or displacement, passes the peak point. The collapsed position could be visibly defined in this case. However, if the structure is going to collapse by plasticity damage, the collapse is at first defined by appearing at the full-yield section without the peak. When a section is fully yielded, to satisfy the yield law, no additional internal forces can be generated in the section. However, due to the load step increment, the axial force of each section is subsequently increased. Hence, after fully yielding, the moments at each section shall be reduced regardless of the effect of hardening patterns. For this reason, the full yield point is defined as the peak point of moments, as depicted in Figure

4.3 *Results*

4.3.1 *Linear redundancy*

Figure 8shows the plotted results of the conventional linear redundancy method for one representative case, Case 1. The R index was calculated from members around the damage mainly caused by the virtual break. Similarly, Figure shows the results from the proposed method for Case 1. It demonstrates that both the conventional method and the proposed method indicate that the diagonal D2 is the critical member that suffers the most from the virtual break in Case 1. The other cases were treated by the same principles. Figures 12 to 17 show the plotted the curves of the R-value versus the release load factor, α, of the critical members in six cases of study. The locations of the critical members are exhibited inside these figures. The region R>1 indicates that the critical member has failed.

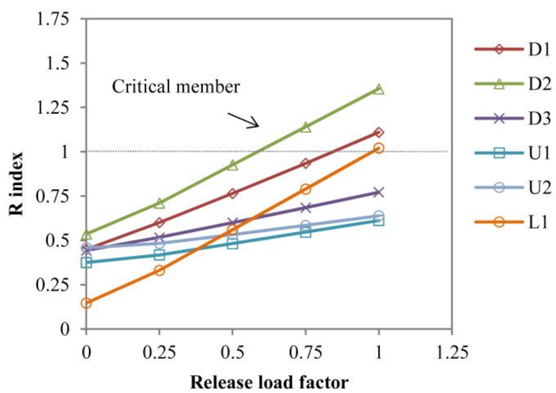


Figure 8. Conventional linear redundancy index in Case 1

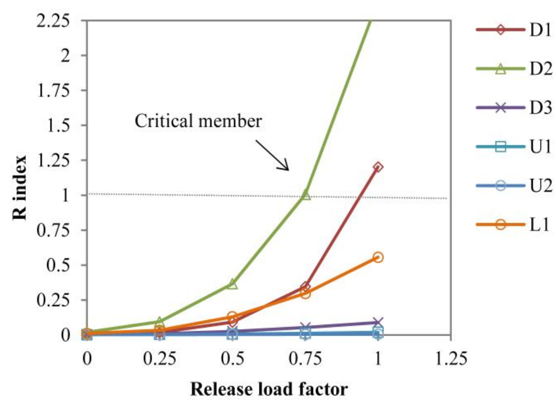


Figure 9. Proposed redundancy index in Case 1

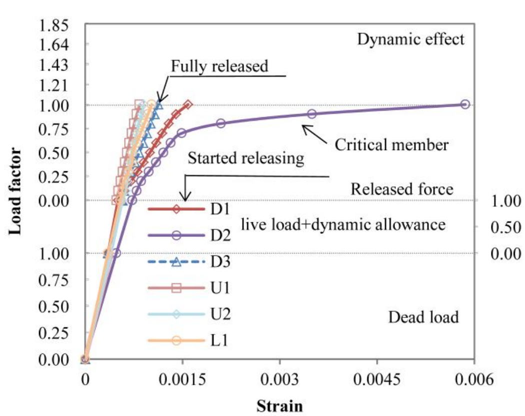


Figure 10. Strain of members around the virtual break in Case1

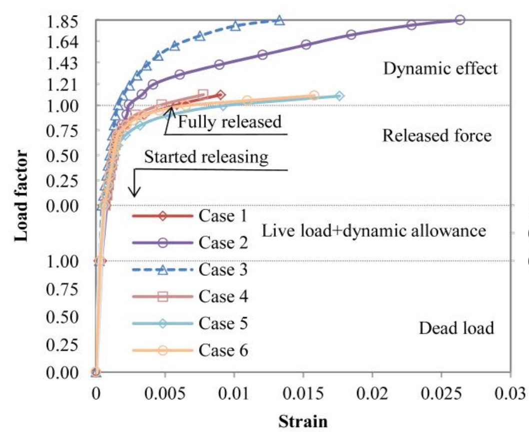


Figure 11. Strain-load curves at critical members in all cases

4.3.2 *Nonlinear redundancy*

Member strain was also observed. Figure shows the plotted strain of members around the virtual break in Case 1. It shows that the diagonal D2 is the critical member after the virtual break in Case 1 according to the nonlinear analysis. Figure 11shows the strain curves of critical members of all cases during nonlinear redundancy analysis. The observed positions are the largest strain locations in the bridge. These positions are indicated in details from Figure to Figure . It is obvious that no peak of the curve exists on the response of the bridge. This suggests that the studied cases will collapse due to plasticity. For this reason, it is necessary to investigate the moment variations of members to determine the full plastic point. The moment curves of the critical members are plotted in Figure to Figure . On the other hand, the linear redundancy analysis looked for the point at which the bridge violated the safety criteria. The safety index, R, of all members, accounts for cases in which the release load factor, α, ranges from 0 to 1.854. Figure to Figure show the plotted curves of R-values versus load factors (α) of the members with the highest risk of sudden member break. The nonlinear and linear redundancy analyses show the same high-risk members for all cases of study. From these figures, the collapse points in the redundancy analysis methods are defined.

4.4 *Comparisons and Discussion*

The collapse release load factor in the redundancy methods is calculated from Figure to Figure . Figure summarizes the value of this collapse load factor. Figure depicts the accuracy of the conventional linear method and the proposed nonlinear redundancy method. The accuracy is computed only by dividing the collapse release load factor in the conventional method and the proposed method by the value defined in the nonlinear method. It is shown that the conventional linear redundancy method yields the first violation of safety criteria at load factor of approximately 70% of the collapse load factor in the nonlinear redundancy method, except for Cases 5 and 6. On the other hand, the proposed method always yields over 90% accuracy in all cases. In Cases 5 and 6, the bridge is going to collapse due to the plasticity of the compressive members. Most recent research shows that the plastic strength of columns is a linear rather than a curved line, as in tensile members. The conventional linear method employs a linear line to express the strength of the compressive members. It is the reason that the conventional redundancy method results are highly accurate in these two cases. This is also the reason that the proposed method maintains the compressive member strengths in cases of uniaxial bending, as in the conventional method of Equations (15) and (16), only with a developed new relation when they are combined in biaxial bending cases.

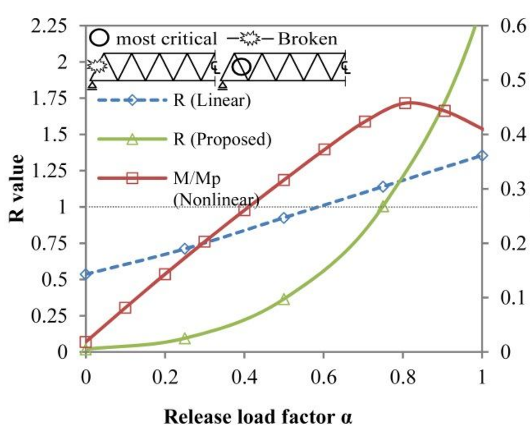


Figure 12. Comparison in Case 1

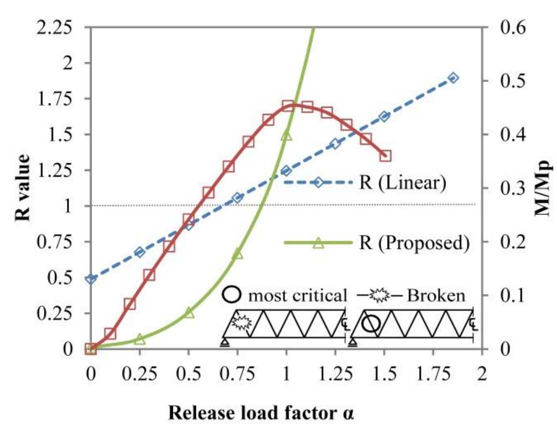


Figure 13. Comparison in Case 2

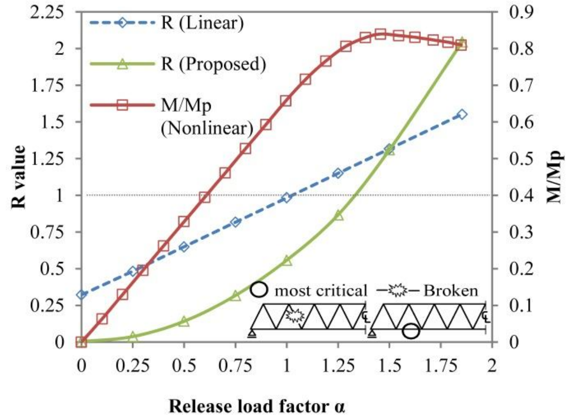


Figure 14. Comparison in Case 3

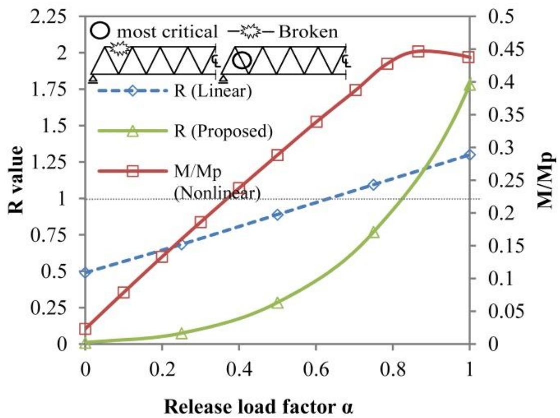


Figure 15. Comparison in Case 4

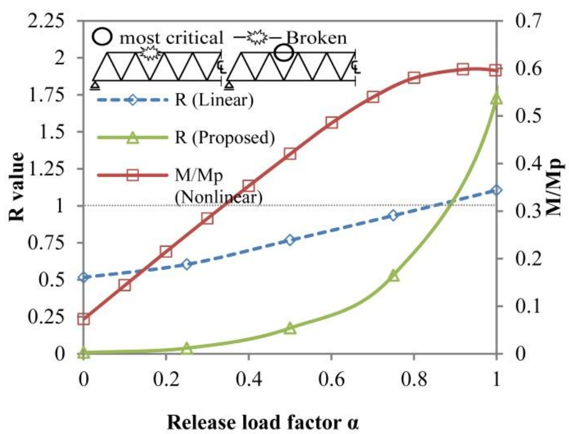


Figure 16. Comparison in Case 5

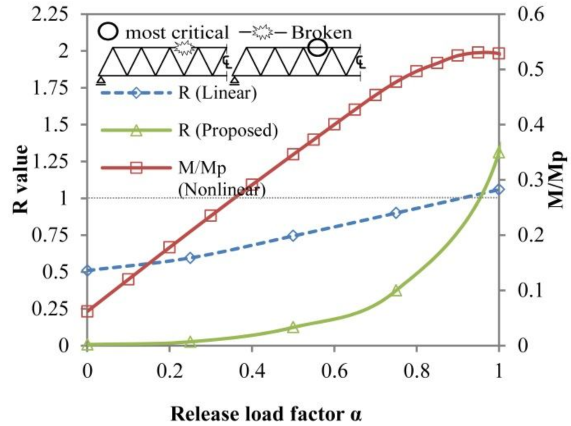


Figure 17. Comparison in Case 6

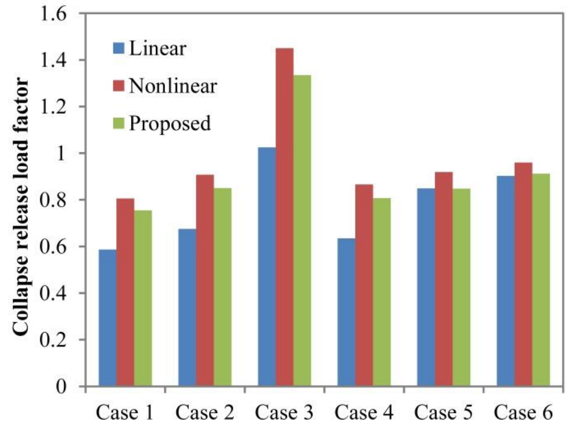


Figure 18. Release load factor at collapse

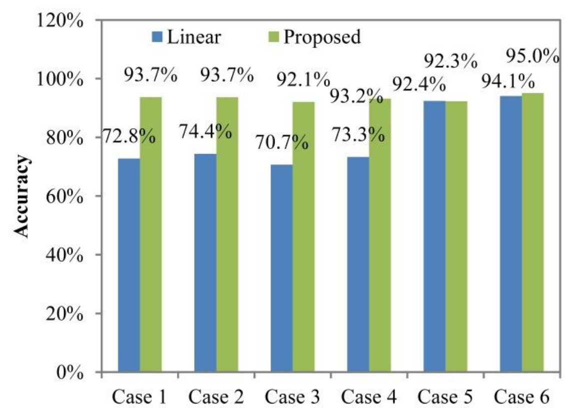


Figure 19. Comparison to the nonlinear method

5 CONCLUSIONS

In this study, redundancy analysis of steel truss bridges was reviewed. The conventional linear redundancy analysis is simple, but has limited accuracy. This method merely employs a linear equation to express the member strength without the effect of the plastic region. This study developed more accurate equations to represent the plastic strength of the members. The member strength in the proposed method uses nonlinear equations instead of linear equations, as in the conventional method. The effect of the plastic region in most typical sectional shapes, such as a box shape, H shape, and I shape, is considered. Validation with the linear redundancy analysis showed that the new method is consistent with the nonlinear redundancy method. Compared to the nonlinear redundancy analysis method, the conventional method yields an accuracy of approximately 70%, whereas the proposed method is over 90% accurate. The contribution of this study is the development of a new linear redundancy method that is as simple as the conventional one, but more accurate. The proposed method can be applicable to steel truss bridges with typical cross sections of members, such as I shape, H shape and box shape sections. This method can be developed for other types of cross sections with greater accuracy than the convention linear method because it represents a closer strength curve to the real strength curve than the conventional method.

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