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Algorithm to identify axle weights for an innovative BWIM system- Part I

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ABSTRACT: This paper introduces an innovative bridge weigh-in-motion (BWIM) system, which uses instrumented bridge as a large scale to continuously collect vehicle information of passing vehicles, including speed, axle spacing, and axle weights. Based on field test on the bridge on highway I-78 in Alabama, this paper proposes an algorithm for the BWIM system to identify the axle weights of heavy vehicles on highways. The BWIM system takes the influence line (IL) as a reference to calculate axle weights. At first, the algorithm for IL calculation is proposed based on continuously measured bridge response (strain) of two calibration vehicles (5-axle semi-trailer) of known weights and axle spacing running many times across the instrumented bridge. Then the research herein proposes a modified Moses' algorithm based on the calculated IL to calculate axle weights in terms of the least square methods with minimization the differences between measured bridge response (bending moments) and predicted ones at mid span when the vehicle passes the bridge. The mathematical equations to calculate the IL and axle weights are derived, and the proposed algorithm is implemented by computer program in MATLAB. Field testing of a concrete slab-girder bridge on highway I-78 in Alabama in the U.S. is implemented to test and evaluate the accuracy of the proposed algorithm in the identification of the axle weights with the comparison of static weights and that from bending plate WIM (BPWIM) system of moving heavy vehicles. Test results shows that BWIM system exhibits advantages over BPWIM system in acquiring actual live load data owing to its accuracy, portability and cost-effectiveness.

1 INTRODUCTION

In the U.S., about half of the 600,000 highway bridges were built before 1940, which means that they are, on average, about 60 years old. Some 220,000 (40%) of the bridges are considered defective and are eligible candidates for highway bridge replacement and rehabilitation program. One of the crucial factors to optimize safety evaluation of existing old and deficient bridges is based on the acquisition of real live load data for heavy vehicles on highways. Bridge weigh-in-motion (BWIM) systems exhibit advantages over traditional pavement weigh-in-motion (WIM) systems in acquiring actual live load data owing to their portability and cost-effectiveness.

The BWIM system we presented herein takes the influence line (IL) as a reference to calculate axle weights. IL of the bridge is one of the most crucial parameters in the application of BWIM system as they describe bridge behavior under the moving load. For the practical and commercial BWIM system, Moses' algorithm is widely used and needs calibration vehicles of known weight and axle spacing to calculate actual IL coordinates based on measured bridge response. Therefore, in order to improve the accuracy of axle weight calculation, the first important issue is to improve the algorithm of IL calculation based on the acquired bridge response measurement, so as to make the calculated IL represent the actual bridge behavior.

Zag (2005) develops commercial BWIM system based on Moses' theory (Moses, 1979). The theoretical IL of the system is adjusted and revised to achieve better conformity with the actual situation to acquire better results of axle weights and gross vehicle weight (GVW) through the response to a calibration truck of known weight passing the bridge (Žnidarič et al., 2002). McNulty & O'Brien (2003) describe a 'point-by-point' graphical method of experimentally deriving the IL from the bridge response to a calibration truck. O'Brien et al. (2006) propose a mathematical method to derive the IL from direct measurements of the load effect in response to a vehicle of known weight and axle spacing.

Based on the field test of a concrete slab-girder bridge on highway I-78 in Alabama in the U.S., the research herein proposes a method to calculate IL based on the measured bridge response which is acquired from the data acquisition system with the commercial BWIM system as a high rate of sampling (512 sampling per second). The algorithm of IL calculation is based on least square methods with minimization the differences between measured and modeled response (strains) at mid span when pre-weighed calibration vehicles pass the bridge.

After the actual IL is obtained, the next critical issue is to improve algorithm to obtain better accuracy of calculation for axle weights. Many researchers demonstrate that the axle weight calculation based on Moses' algorithm needs to solve a set of ill-conditioned equations (Rowley et al., 2008, González et al., 2008, O'Brien et al., 2009). The ill-conditioned nature of the problem makes it difficult to distinguish individual axle loads within closely spaced axle combinations such as tandems, semi-trailers, and trailers. They also investigate that the regularization technique significantly improves the accuracy by adding a regularization term on the original problem and finding an optimum regularization parameter.

The research herein is not to investigate the effectiveness of regularization technique, but rather to study the effectiveness of the proposed modified Moses' algorithm based on the calculated IL. The proposed algorithm is applicable for the typical slab-girder bridge in the U.S. in the identification of axle weights and GVW with the comparison of static weights of moving heavy vehicles. The mathematical equations to calculate ILs and axle weights are derived, and the proposed algorithm is implemented by computer program written by the author in MATLAB.

The proposed algorithm of this research will consider two different conditions. For the first condition, the whole bridge is considered as a single beam and each girder is assumed to have the same modulus section (Z) and modulus of elasticity (E) so that the whole bridge just has one IL; for the second one, different girders are considered to have different properties (E and Z). The proposed algorithm for the first condition is presented in this paper (Part I) and that for the second one is presented in the following paper named as 'Algorithm to Identify Axle Weights for an Innovative BWIM system- Part II'.

2 SYSTEM AND INSTALLATION OF SENSORS

2.1 *system*

The concept of BWIM system was developed by Moses and his team in 1979 (Moses, 1979, Moses & Verma, 1987). This method uses instrumented bridge as a large sensor, and the transducers are mounted on the soffit of each girder along a line parallel to the longitudinal direction of the bridge to obtain axle weight and GVW of heavy trucks passing the bridge (Moses, 1979, Cost, 1999). In recent years, advanced BWIM systems have shown remarkable potential in detecting oversized and overweight commercial trucks in Europe (Zag, 2005).

In detecting vehicles, most of the current, conventional BWIM systems require two axle or vehicle detectors installed on the pavement of each lane of interest to provide vehicle silhouette and velocity. For temporary installations, pneumatic tubes or tape switches are widely used, but the durability is poor and it is not safe for personnel working near traffic. Piezoceramics sensors are much more durable but are more expensive, and the installation requires lane closures. Nowadays, innovative BWIM system replaces traditional ones with axle detector technology named like 'nothing on the road (NOR)' or 'free of axle detector (FAD)' (Zag, 2005). This technology requires additional transducers mounted underneath the bridge slab to induce signals of the passing vehicles so as to detect them. For this type of BWIM system, the advantage is that it totally eliminates all actions on the pavement and consequently reduces costs of installation and inconveniences to road users without interfering with the traffic flow. In addition, as all the equipment is hidden under the bridge, all the detection action is totally invisible to truckers and it is particularly effective in terms of maintenance, especially in harsh climates. Most important of all, the whole system is portable and can be reused for many bridges so that the whole cost of the measuring system will be significantly reduced in comparison with pavement WIM system (Zag, 2005).

The BWIM system described herein is a FAD system. The main part of the BWIM system includes: (1) sensors to acquire the signals, including weighing sensors and FAD sensors. The former is mounted on the soffit of each girder along a line normal to the direction of the bridge to weigh axle loads and GVW, the latter is installed right under the slab to detect the vehicle axles to obtain information of axle space, speed and etc.; (2) cabinet to keep the processor of the system (electronics in the casing and cabling, computer and software); (3) antenna, personal digital assistant (PDA) and wireless fidelity (Wi-Fi) system to represent the core of the BWIM system to communicate with each other through the transmission control protocol/internet protocol (TCP/IP); (4) camera system to recognize and capture pictures of vehicles; and (5) solar panels to provide power supply. The components of the BWIM system we used herein are illustrated in Figure 1.

(1) FAD sensors; (2) Spider; (3) Weighing sensors; (4) Cabinet & panel; (5) Batteries housing; (6) Solar panels; (7) Solar panel installation; (8) Antenna; (9) Camera; (10) PDA

Figure 1. Components of the BWIM system

2.2 *Sensor installation*

The bridge selected for the BWIM installation is located on the highway I-78 East in Graysville, Alabama, three miles west of highway I-22. The number of the bridge is BIN 7633. The bridge is smooth on the joint, and the approach to the bridge is even. The bridge is a three span simple supported T-beam bridge with span $3 \times 42\, ft(12.8m) = 126\, ft(38.4m)$ with two lanes in each direction. Figure 2 shows the overview of the instrumented bridge with BWIM system. Figure 3 illustrates the sensor position of the bridge.

Figure 2. Overview of bridge on highway I-78

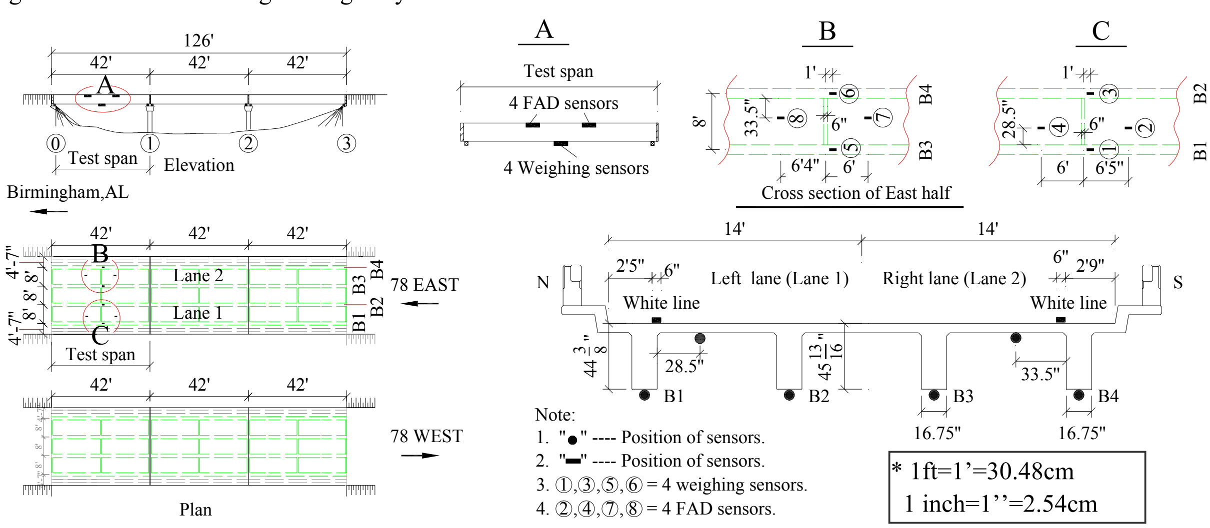

Figure 3. Sensor positions of bridge on highway I-78

For the instrumentation of the BWIM system, the end span to the city of Birmingham direction was selected as test span (Fig. 3). Four weighing sensors were mounted longitudinally on the soffit of concrete girders (one sensor for each girder) with one foot off the centre because of the diaphragm. To detect the vehicles and acquire the axle number of vehicle and axle spacing, four FAD sensors were mounted longitudinally underneath the concrete slab 12 feet apart for each separate lane.

3 INITIAL CALIBRATION OF THE BWIM SYSTEM

The initial calibration test for the BWIM system was conducted on Nov 18, 2008. The initial calibration test was calibrated under the test condition (R1-I) according to the European specifications for WIM (Cost, 1999). As it was observed that the representative vehicle of highway I-78 was semi-trailer, the initial calibration was conducted with two semitrailers loaded to a capacity of 80,000 lbs (36,287kg) from Alabama Department of Transportation (ALDOT) as pre-weighed trucks. The following table 1 provides details of the calibration vehicles.

Table 1. The initial calibration vehicle information of the bridge on highway I-78

Vehicle No.	Axle weight (lb)						Axle distance (inch)			
	GVW	1st axle	2nd axle	3rd axle	4th axle	5th axle	A1-A2	A2-A3	A3-A4	A4-A5
1	79,000	11,050	15,650	16,100	18,200	18000	170	53	440	51
2	78,200	10,050	16,000	15,800	18,300	18050	171	53	438	50

* 1 lb= 0.454 kg, 1 inch=2.54 cm

During the initial calibration test, two pre-weighed trucks were running with different speeds at different lanes. The total runs were 24 runs (each lane with 12 runs). During the whole initial calibration, we missed 1 run in each lane, and we had multiple presences for one time. Hence, we have 10 runs for each lane. Figure 4 shows the pictures of the calibration vehicles.

Static photos (Left: No. 1 right: No. 2) Photos from BWIM system (Left: lane 1 Right: lane 2)

Figure 4. Vehicles for the initial calibration of bridge on highway I-78

4 ALGORITHM OF IL CALCULATION AND FIELD TEST VERIFICATION

4.1 *Algorithm of IL calculation*

For a static vehicle at a certain location on a girder bridge, the total longitudinal gross bending moment at a specific bridge section can be expressed as a function of time, and can be defined by summing all the individual girder moments. For each girder, at time step *k*, the bending moment of girder *i*, M_k^i, equals:

$$M_k^i = E_i Z_i \varepsilon_{ik}^t \tag{1}$$

where M_k^i = the bending moment of girder i; E_i and Z_i = the section modulus and the modulus of elasticity of the ith girder, respectively; and ε_{ik}^t = the predicted theoretical strain at time step k at the soffit of the ith girder.

Each girder is assumed to have the same E and Z, say, $E_i Z_i = EZ$. For this bridge, we assume that all the girders has the same properties as exterior girder. Then, the total bending moment across the bridge section, M_k, at time step k is given by equation

$$M_k = EZ\sum_{i=1}^{g}\varepsilon_{ik}^t = EZ\varepsilon_k^t \tag{2}$$

where M_k = the gross bending moments at mid span at time step k; and ε_k^t = the sum of the predicted theoretical strains at all girders at time step k.

When a calibration truck with N known axle weights, P_1, P_2, ..., P_N, passes the bridge, at each time step k, the corresponding theoretical load effect (bending moment, M_k) caused by the calibration truck, is given by

$$M_k = \sum_{i=1}^{N} P_i I_{(k-C_i)} \tag{3}$$

The corresponding theoretical bridge response (theoretical strain, ε_k^t) caused by the calibration truck at time step k is given by

$$M_k = EZ\varepsilon_k^t = \sum_{i=1}^{N} P_i I_{(k-C_i)} \tag{4}$$

$$\varepsilon_k^t = \frac{1}{EZ}\sum_{i=1}^{N} P_i I_{(k-C_i)} \tag{5}$$

$$C_i = \frac{D_i f}{v} \tag{6}$$

where $I_{(k-C_i)}$ = the IL ordinate of the ith axle at time step k; f = the scanning frequency of a high rate of data acquisition sampling system; v = the vehicle velocity; D_i = the distance between axle i and the first axle; and C_i = the number of scans corresponding to D_i ($C_1 = 0$). The details are illustrated with a three-axle vehicle and shown as Figure 5.

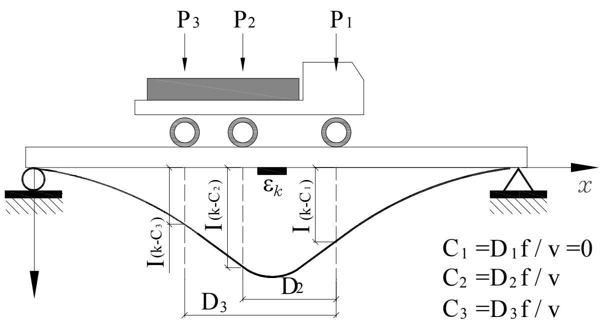

Figure 5. IL ordinates of calibration vehicle at time step *k*

During the whole calibration, we assume that vehicle velocity is constant. Vehicle speed can be determined by the two FAD sensors of each lane mounted under the slab of the bridge at $L/4$ and $3L/4$, respectively.

From time step 1 to K, say, from the moment the first axle reach the position prior to the bridge (the starting-point for the IL, recorded as 'a' scan) to the moment the last axle leave the position posterior to the bridge (recorded as 'b' scan), we will have K scans of the strain data, where 'a' means the instant the first axle reaches a specified point prior to the bridge, and 'b' means the instant the last axle leaves a specified point posterior to the bridge. There is no need to know the exact position at which the applied load causes the bridge to start bending. Therefore the uncertainty surrounding the real boundary conditions and the small strains generally induced near the supports are avoided (González & O'Brien, 2002).

Based on the least square method, an error function between the measured bridge response (sum of strain) and the theoretical bridge response will be defined as

$$E = \sum_{k=1}^{K} (\varepsilon_k^m - \varepsilon_k^t)^2 \tag{7}$$

where ε_k^m is the sum of measured strains at all girders at time step k.

We use differential calculus to differentiate E with respect to the set of influence ordinates I_k, and set the expression equal to zero. For simplification, we use three-axle calibration truck as an example (Fig 5.) to derive IL ordinates. In this case, the theoretical bridge response (strain) under the calibration truck at time step k is

$$\varepsilon_k^t = \frac{1}{EZ}[(P_1 I_{(k-C_1)} + P_i I_{(k-C_2)} + P_3 I_{(k-C_3)})] \qquad k = 1,\cdots,K \tag{8}$$

From scans of data acquisition 1 to K, the corresponding theoretical bridge response (strain) under different scans is as:

$$E = \sum_{k=1}^{K} (\varepsilon_k^m - \varepsilon_k^t)^2 \tag{9}$$

In the error function, items relating to I_R $(R = 1, \cdots, K - C_3)$ are ε_R^t, $\varepsilon_{R+C_2}^t$, and $\varepsilon_{R+C_3}^t$. Differentiating E with respect to set of influence ordinates I_R and minimizing E, the set of influence ordinates will make the partial derivatives are zero.

$$\frac{\partial E}{\partial I_R} = 0 \qquad (R = 1, \cdots, K - C_3) \tag{10}$$

At scan $R = a + 1$, the front axle of the vehicle approaches the bridge; at scan $R = a + C_3$, the last axle of the vehicle approaches the bridge; at scan $R = K - b - C_3$, the first axle of the vehicle leaves the bridge, and at scan $R = K - b$, the last axle of the vehicle leaves the bridge. When $(C_3 + a) < R < (K - b - C_3)$, it is the general case when all axles are on the bridge structure.

Summing all the above $K - C_3$ equations from time step $R = 1, \cdots, K - C_3$, the equations are written in a matrix form as:

$$[W]_{(K-C_3)\times(K-C_3)} \times \{I\}_{(K-C_3)\times 1} = \{\varepsilon\}_{(K-C_3)\times 1} \tag{11}$$

where $\{I\}$ = an IL ordinates vector; $\{\varepsilon\}$ = a vector dependent on the axle weights of the vehicle and the measured load response (strain), the element of the vector at row R is $\varepsilon_R = EZ(P_1\varepsilon_R^m + P_2\varepsilon_{R+C_2}^m + P_3\varepsilon_{R+C3}^m)\, R = 1, \cdots, K - C_3$; and $[W]$ = a sparse symmetric matrix dependent on the vehicle axle weights listed as following.

$$[W]_{(K-C_3)\times(K-C_3)} =$$

$$\begin{bmatrix}
\sum P_i^2 & 0 & 0 & \cdots & P_2P_1 & 0 & \cdots & P_3P_2 & 0 & \cdots & P_3P_1 & 0 & \cdots & & & & & \\
\cdots & \sum P_i^2 & 0 & 0 & \cdots & P_2P_1 & 0 & \cdots & P_3P_2 & 0 & \cdots & P_3P_1 & 0 & \cdots & & & 0 & \\
\ddots & \cdots & \ddots & \ddots & \ddots & \cdots & \ddots & \ddots & \cdots & \ddots & \ddots & \cdots & \ddots & \ddots & \cdots & & & \\
P_1P_2 & 0 & \cdots & \sum P_i^2 & 0 & 0 & \cdots & P_2P_1 & 0 & \cdots & P_3P_2 & 0 & \cdots & P_3P_1 & 0 & \cdots & & \\
\cdots & P_1P_2 & 0 & \cdots & \sum P_i^2 & 0 & 0 & \cdots & P_2P_1 & 0 & \cdots & P_3P_2 & 0 & \cdots & P_3P_1 & 0 & \cdots & \\
\ddots & \cdots & \ddots & \ddots & \cdots & \ddots & \ddots & \ddots & \cdots & \ddots & \ddots & \cdots & \ddots & \ddots & \cdots & \ddots & \ddots & \cdots \\
P_2P_3 & 0 & \cdots & P_1P_2 & 0 & \cdots & \sum P_i^2 & 0 & 0 & \cdots & P_2P_1 & 0 & \cdots & P_3P_2 & 0 & \cdots & P_3P_1 & 0 \\
\cdots & P_2P_3 & 0 & \cdots & P_1P_2 & 0 & \cdots & \sum P_i^2 & 0 & 0 & \cdots & P_2P_1 & 0 & \cdots & P_3P_2 & 0 & \cdots & P_3P_1 \\
\ddots & \cdots & \ddots & \ddots & \cdots & \ddots & \ddots & \cdots & \ddots & \ddots & \ddots & \cdots & & \ddots & \cdots & \ddots & \ddots & \cdots \\
P_1P_3 & 0 & \cdots & P_2P_3 & 0 & \cdots & P_1P_2 & 0 & \cdots & \sum P_i^2 & 0 & 0 & \cdots & P_2P_1 & 0 & \cdots & P_3P_2 & 0 \\
 & P_1P_3 & 0 & \cdots & P_2P_3 & 0 & \cdots & P_1P_2 & 0 & \cdots & \sum P_i^2 & 0 & 0 & \cdots & P_2P_1 & 0 & \cdots & A_3A_2 \\
 & & \ddots & \ddots & \cdots & \ddots & \ddots & \cdots & \ddots & \ddots & \cdots & \ddots & \ddots & \ddots & \cdots & \ddots & \ddots & \cdots \\
 & & & P_1P_3 & 0 & \cdots & P_2P_3 & 0 & \cdots & P_1P_2 & 0 & \cdots & \sum P_i^2 & 0 & 0 & \cdots & P_2P_1 & 0 \\
 & & & & \ddots & \ddots & \cdots & \ddots & \ddots & \cdots & \ddots & \ddots & \cdots & \ddots & \ddots & \ddots & \cdots & \ddots \\
 & 0 & & & & P_1P_3 & 0 & \cdots & P_2P_3 & 0 & \cdots & P_1P_2 & 0 & \cdots & \sum P_i^2 & 0 & 0 & \cdots \\
 & & & & & & \ddots & \ddots & \cdots & \ddots & \ddots & \cdots & \ddots & \ddots & \cdots & \ddots & \ddots & \ddots \\
 & & & & & & & P_1P_3 & 0 & \cdots & P_2P_3 & 0 & \cdots & P_1P_2 & 0 & \cdots & \sum P_i^2 & 0 \\
 & & & & & & & & P_1P_3 & 0 & \cdots & P_2P_3 & 0 & \cdots & P_1P_2 & 0 & \cdots & \sum P_i^2
\end{bmatrix} \tag{12}$$

From matrix $[W]$, we can find that the main diagonal of the matrix is the sum of the squares of axle weights. For a three-axle calibration vehicle, the main diagonal is

$$W_{i,i} = \sum_{i=1}^{3} P_i^2 = P_1^2 + P_2^2 + P_3^2 \quad i = 1, \cdots, K - C_3 \tag{13}$$

The upper triangle elements are given by

$$\begin{aligned}
\mathrm{W}_{\mathrm{i,i}+(C_3-C_2)} &= P_2P_3; \qquad i + (C_3 - C_2) \le K - C_3 \\
\mathrm{W}_{\mathrm{i,i}+(C_2-C_1)} &= P_1P_2; \qquad i + C_2 \le K - C_3 \\
\mathrm{W}_{\mathrm{i,i}+(C_3-C_1)} &= P_1P_3; \qquad i + C_3 \le K - C_3
\end{aligned} \tag{14}$$

The other numbers in this matrix are zeros. The corresponding lower triangular elements are symmetric.

The IL ordinates are calculated through the calibration vehicle passing the bridge.

$$\{I\}_{(K-C_3)\times 1} = [W]^{-1}{}_{(K-C_3)\times(K-C_3)} \times \{\varepsilon\}_{(K-C_3)\times 1} \tag{15}$$

The above equations are special for 3-axle vehicles. For vehicles with different number of axles, similar equations can be derived. If the calibration vehicle has N axles, the matrix equation will be:

$$[W]_{(K-C_N)\times(K-C_N)} \times \{I\}_{(K-C_N)\times 1} = \{\varepsilon\}_{(K-C_N)\times 1} \tag{16}$$

$$\{I\}_{(K-C_N)\times 1} = [W]^{-1}{}_{(K-C_N)\times(K-C_N)} \times \{\varepsilon\}_{(K-C_N)\times 1} \tag{17}$$

where $\{\varepsilon\}$ in equation (17) is similar to that in equation (11), the element of the vector at row R is $\varepsilon_R = EZ(P_1\varepsilon_R^m + P_2\varepsilon_{R+C_2}^m + P_3\varepsilon_{R+C3}^m + \cdots P_N\varepsilon_{R+C_N}^m)\ R = 1, \cdots, K - C_3$.

4.2 *Field test verification of IL calculation*

The IL was calculated based on the measured strain data from field test on the bridge on highway I-78 in Alabama. The calibration vehicle was five-axle semi-trailer (table 1). In order to verify the proposed algorithm in IL calculation, 10 runs on lane 1 were calculated. Vehicle No. 1 is the calibration vehicle which is corresponding to run 1, 2, 3, 4 and 5; vehicle No. 2 is corresponding to run 6, 7, 8, 9 and 10.

Figure 6 illustrates the calculated IL for run 1 based on the above equations; and Figure 7 displays the comparison of measured strains and predicted ones from the calculated IL. Figure 7 shows an excellent match between measured strains and predicted ones using the calculated IL, which illustrates the accuracy and effectiveness of the proposed method to calculated IL.

In order to apply the calculated ILs to represent the behavior of the actual condition, usually we need to average the ILs of all runs, or average some selected runs. Figure 8 illustrates the calculated ILs for all runs and Figure 9 shows the results of average ILs considering different number of repeated runs.

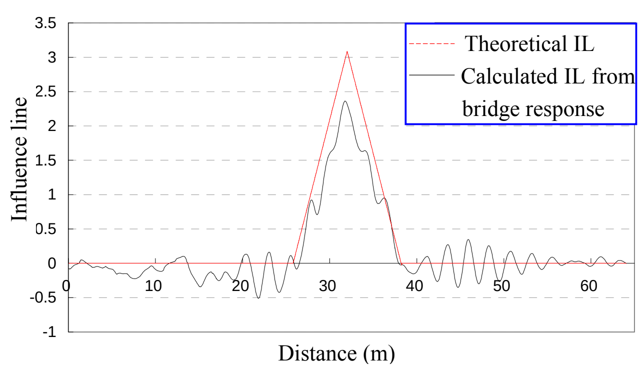

Figure 6. Calculated IL for run 1

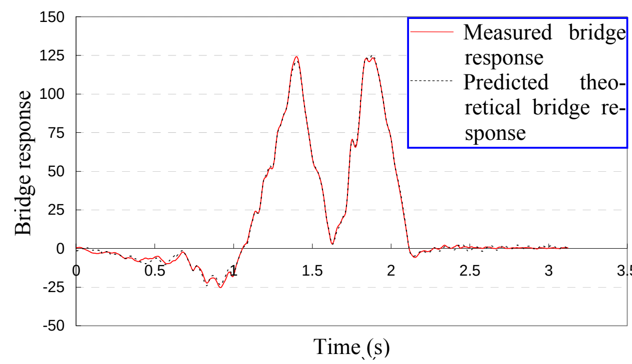

Figure 7. Measured and predicted response of run 1

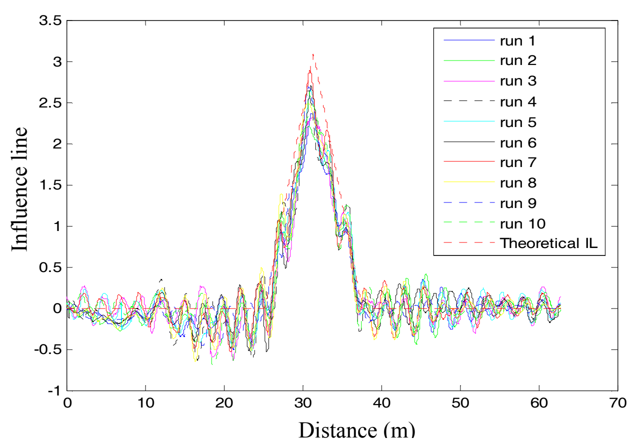

Figure 8. Calculated ILs of all runs

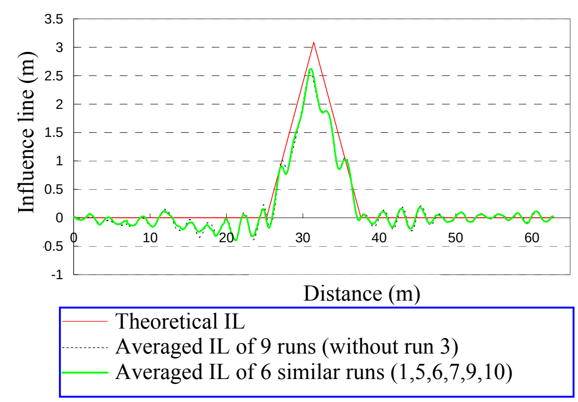

Figure 9. Average of ILs considering different runs

From Figure 8, we can see that the IL obtained from run 3 is different from other runs, and we also identify that the measured strains appear slightly difference from the predicted strains in comparison with other runs. Thus, when we consider the calculated IL as reference for axle weight calculation, we should neglect run 3. From Figure 9, we can see that the averaged ILs of 9 runs is close to the averaged ILs of 6 selected runs. From another point of view, it demonstrates the effectiveness and stability of the proposed method to calculate IL.

5 ALGORITHM OF CALCULATION OF AXLE WEIGHTS AND FIELD TEST VERIFICATION

5.1 *Algorithm of axle weight identification*

When a vehicle passes the bridge with the strain of the sensors measured continuously, gross bending moment can be expressed as a function of time and formulated by summing all the individual girder moments. Considering all the girders are identical, at time step k, the measured gross bending moments at mid span, M_k^m, equals to the summation of the bending moment of all the girders.

$$M_k^m = \sum_{i=1}^{g} M_k^i = EZ\sum_{i=1}^{g} \varepsilon_{ik}^m = EZ\varepsilon_k^m \tag{18}$$

where M_k^m = the measured gross bending moments at mid span; M_k^i = the measured bending moment of the ith individual girder; $i = 1,\cdots,g$ (number of girders); ε_k^m = the sum of the measured strains at all girders at time step k; and ε_{ik}^m = the measured strain at time step k at the soffit of the ith girder.

During the passing of vehicles, if the vehicle has the number of axles, N, the theoretical number of unknowns for each vehicle will be N. During the truck passage at different time steps $k = 1,\cdots,K\,(K > N)$, a set of K equations will be obtained for N unknowns. Then the measured bending moments, M_k^m, are compared to the modeled bending moments.

Figure 10 gives as an example of the bending moment equation on a simply supported bridge at time step k, when the first axle is x from the support. With the axle spacing $D_1, D_2, \cdots, D_N$ and the bending moment IL with I_k^i ($k = 1,\cdots,K$, $i = 1,\cdots,N$), the predicted theoretical bending moment, M_k^t, can be expressed as:

$$M_k^t = EZ\sum_{i=1}^{g} \varepsilon_{ik}^t = EZ\varepsilon_k^t = P_1 I_k + P_2 I_{(k-C_2)} + \cdots + P_3 I_{(k-C_N)} = P_1 I_k^1 + P_2 I_k^2 + \cdots + P_N I_k^N \tag{19}$$

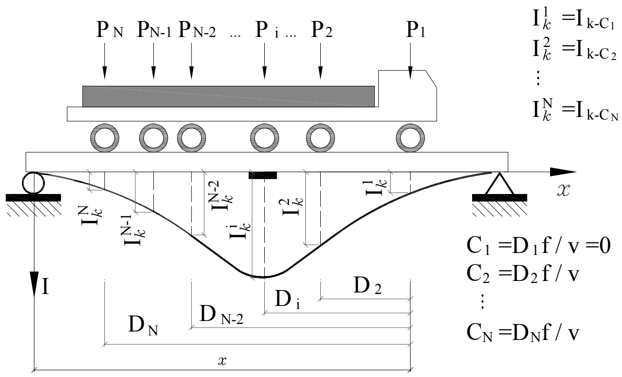

Figure 10. Bending moments at time step k

Then the predicted theoretical strain is:

$$\varepsilon_k^t = \frac{1}{EZ}(P_1 I_k^1 + P_2 I_k^2 + \cdots + P_N I_k^N) \tag{20}$$

where M_k^t = the predicted bending moment at the sensor location; I_k^i = the IL ordinates of the total bending moment for the ith axle at a particular point at time step k; and ε_k^t = predicted theoretical strain (sum of strain of the ith girder, ε_{ik}^t, ($i = 1,\cdots,g$)) at time step k.

Moses' algorithm on BWIM concept is based on the comparison of measured and modeled bridge response at mid span, and he defines an error function between measured gross bending moments and predicted bending moments based on a theoretical IL (Moses, 1979). However, the theoretical IL can not sufficiently represent the actual bridge behavior. The IL obtained from measured strain data will be closer to the actual condition. According to the afore-mentioned algorithm, we can obtain the corresponding IL based on calibration trucks passing the bridge.

From equation (20), for time steps $k = 1,\cdots,K$, we can get K numbers of predicted strain:

$$
\begin{Bmatrix} \varepsilon_1^t \\ \varepsilon_2^t \\ \varepsilon_3^t \\ \vdots \\ \vdots \\ \varepsilon_K^t \end{Bmatrix}_{K\times 1} = \frac{1}{EZ} \begin{bmatrix} I_1^1 & I_1^2 & \cdots & \cdots & I_1^N \\ I_2^1 & I_2^2 & \cdots & \cdots & I_2^N \\ I_3^1 & I_3^2 & \cdots & \cdots & I_3^N \\ \vdots & \vdots & & & \vdots \\ \vdots & \vdots & & & \vdots \\ I_K^1 & I_K^2 & & & I_K^N \end{bmatrix}_{K\times N} \begin{Bmatrix} P_1 \\ P_2 \\ \vdots \\ \vdots \\ P_N \end{Bmatrix}_{N\times 1} \tag{21}
$$

where K is the number of scans of strain readings, and in a simple form we have

$$
\{\varepsilon^t\}_{K\times 1} = [IL]_{K\times N}\{P\}_{N\times 1} \tag{22}
$$

where $\{\varepsilon^t\}$ = the vector of theoretical static strain; $[IL]$ = the matrix of influence ordinates; and $\{P\}$ = the vector of axle weights to be determined.

Assuming an error function E, which is the squares of the difference between the theoretical and measured strains; thus, the problem can be solved by minimizing the error function.

$$
E = \sum_{k=1}^{K} (\varepsilon_k^t - \varepsilon_k^m)^2 \tag{23}
$$

In matrix form, the error function can be written as:

$$
E = \{\{\varepsilon^m\} - \{\varepsilon^t\}\}^T \{\{\varepsilon^m\} - \{\varepsilon^t\}\} \tag{24}
$$

$$
E = \{\varepsilon^m\}^T\{\varepsilon^m\} - \{\varepsilon^m\}^T[IL]\{P\} - \{P\}^T[IL]^T\{\varepsilon^m\} + \{P\}^T[IL]^T[IL]\{P\} \tag{25}
$$

Minimizing the error function with respect to the vector of axle weights results in:

$$
\frac{\partial E}{\partial\{P\}} = 0 - [IL]^T\{\varepsilon^m\} - [IL]^T\{\varepsilon^m\} + 2[IL]^T[IL]\{P\} = 0 \tag{26}
$$

$$
\{P\} = \left[[IL]^T[IL]\right]^{-1}[IL]^T\{\varepsilon^m\} \tag{27}
$$

From this equation, the axle loads can be obtained, and the GVW can be obtained by summing axle weights as

$$
\mathrm{GVW} = \sum_{i=1}^{N} P_i \tag{28}
$$

5.2 *Field test verification of axle weight calculation*

Based on the calculated ILs of the 10 runs on lane 1, we employ the calculated ILs to calculate the corresponding axle weight (the averaged ILs of 9 runs listed in Figure 9). Zhao (2010) demonstrate that the scan numbers of measured strain data should cover the whole process of vehicle passing the instrumented bridge in order to improve the accuracy of axle weight calculation. Herein, we choose the case that adding 100 sampling (about $100/512 = 0.2s$) before the vehicle approaches the bridge and after the vehicle leave the bridge to cover the process of the abrupt change of the vehicle approaching or leaving the bridge. Table 2 summarizes the calculated results of all the ten runs.

Fortunately, an existing BPWIM system operated by ALDOT is 4 miles west of the bridge, which is one of ALDOT's 11 pavement WIM sites. For each run of the trucks weighed during the research, the results calculated by the proposed algorithm for BWIM system can be compared to the BPWIM system weight measurements. This allows researchers to demonstrate the relative accuracy of BWIM system and BPWIM system. The test results of BPWIM system is also listed in table 2. For simplicity, we just illustrate the GVW results for BPWIM system. In table 2, 'A1-A5' means the axle number of calibration vehicle from the front to the rear; (2) 'GVW' means the gross vehicle weight; (3) 'SA' means the single axle; (4) 'GOA' means the group of axles; (5) For the mean and standard deviation (st. dev.) of A1, A2, A3, A4, A5, SA and GVW, the number is 10, while that for GOA is 20.

Table 2 shows that the accuracy of the proposed algorithm in axle weight identification is effective and accurate. The percentage error of each single axle weight is less than 20%, that for GOA is less than 6%, and that for GVW less than 7%. The results in table 2 also demonstrate that the accuracy for GVW is acceptable for enforcement screening based on the proposed algorithm in IL calculation and axle weight identification.

Comparisons with static weights on a one-to-one basis for both BWIM predictions and BPWIM measurements have generally fallen below a 10% error for GVW. BWIM results demonstrate considerable repeatability of the predictions with standard errors under 5%. From table 2, we can find that BWIM system can provide better prediction of GVW than BPWIM system. BWIM system has a significant potential in axle weight identification of moving heavy vehicles. BWIM system illustrates significant advantage over BPWIM system owing to its accuracy, portability, and cost-effectiveness.

Table 2 Axle weight comparison with static results of different runs (%)

Item	Proposed algorithm for BWIM system									BPWIM
	A1	A2	A3	A4	A5	SA	GOA		GVW	GVW
Run	(%)	(%)	(%)	(%)	(%)	A1	A2+A3	A4+A5	(%)	(%)
1	-17.4	7.5	-13.5	-11.8	-2.8	-17.4	-3.2	-7.3	-7.1	2.3
2	-8.6	-0.3	-4.4	-5.7	-7.0	-8.6	-2.4	-6.4	-5.1	3.5
3	-12.4	6.9	-11.8	-6.3	-6.7	-12.4	-2.6	-6.5	-5.8	2.9
4	1.2	-9.2	10.8	0.6	-4.1	1.2	0.9	-1.7	-0.3	10.0
5	0.6	2.1	-0.2	-1.9	0.6	0.6	0.9	-0.7	0.2	-2.7
6	7.9	15.4	-12	1.9	3.1	7.9	1.8	2.5	2.9	4.1
7	18.6	5.2	-0.4	12.1	-9.2	18.6	2.4	1.5	4.1	-7.4
8	7.8	6.3	-5.5	3.9	-1.4	7.8	0.4	1.3	1.8	-10.0
9	8.0	-2.1	0.4	14.0	-11	8.0	-0.9	1.6	1.4	-4.9
10	8.7	3.0	-2.2	8.0	-7.5	8.7	0.4	0.3	1.4	-7.0
Mean	1.44	3.48	-3.88	1.48	-4.60	1.44	-0.89		-0.65	-0.92
St. dev.	11.16	6.58	7.35	8.29	4.48	11.16	3.00		3.92	6.41

6 CONCLUSIONS

(1) Based on the comparison of measured bridge response and predicted theoretical ones from the calculated IL, the proposed algorithm in IL calculation is effective and accurate. The calculated IL is closer to the actual IL of the bridge, which represents actual structural behavior and provides data base for the health monitoring of the existing bridges.

(2) For the typical bridge type, simple supported concrete slab-girder bridge, in Alabama and other states in the U.S. as well, the proposed algorithm in axle weight identification is repeatable, effective and accurate.

(3) By the proposed algorithm in axle weight identification, the percentage error of each single axle weight is less than 20%, that for GOA and GVW less than 6% and 7%, respectively.

(4) Field testing of bridge on highway I-78 in Alabama demonstrates that BWIM system can provide better prediction of GVW than BPWIM system by the proposed algorithm based on comparisons with static weights on a one-to-one basis for both BWIM and BPWIM systems.

(5) BWIM system has a significant potential in the application for enforcement screening and illustrates advantage over BPWIM system owing to its accuracy, portability, and cost-effectiveness.

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