

# Influence of longitudinal stiffener on ultimate strength of wide stiffened steel plates: Numerical approach

M.S. Islam & Y. Okui

Saitama University, Saitama City, Saitama 338-8570, Japan

**ABSTRACT:** In design of wide stiffened plates, effects of longitudinal stiffener with large flexural rigidity on ultimate strengths are not yet addressed in Japanese specifications for highway bridges (JSHB). This study investigated the ultimate capacity of stiffened plates with longitudinal stiffeners under uniaxial compression. A wide range of practical relative stiffness parameter of the longitudinal stiffeners has been chosen along with six different width-thickness ratio parameters,  $R_R$  of stiffened plates, to develop a design guideline. Both geometric and material nonlinearity was considered in shell finite element analysis through nonlinear elastoplastic method. Statistical information of initial imperfections i.e. residual stress, whole plate and local plate initial out-of-plane deflections from previous experimental studies were incorporated in numerical simulations. Overall 7 combinations of initial imperfections were employed to account the variability of ultimate strengths. The mean value and standard deviation of ultimate strengths were obtained by the approximate solution procedure. Finally, probabilistic strength information is utilized to calculate lower limit strength (5% fractile) assuming normal distribution of strengths and compared with various design codes. Numerical results manifested that the ultimate capacity of stiffened steel plates increased considerably at relative stiffness value of 3 of the longitudinal stiffeners. For relative stiffness value of 3, rational comparison of ultimate strength with different design codes i.e. JSHB, Eurocode indicates that depending on  $R_R$  values, JSHB underestimates the capacity of stiffened plates in the range of 4% ~ 78%, while Eurocode predicts lower strength for  $R_R \leq 0.6$  and other results fit very well for rest of  $R_R$  values.

## 1 INTRODUCTION

Since the welding technology progresses, the application of steel plated structures increased due to slender, lightweight and fabrication-optimized design criteria. Steel flat plates are usually stabilized with stiffeners; both longitudinally and transversely, resulted improved response of structures (Figure 1). Thus, the behavior of stiffened plates emphasized as a unit rather than individual elements stability. In case of steel box girder, the main concern is to ensure the stability of the thin plate element used for flanges. As the zone of internal supports of a wide stiffened box girder bridges possesses potential in-plane compression, hence the design and stability of wide slender bottom flange dominated by buckling strength of stiffened plates.

The current Japanese specifications of highway bridges predict ultimate buckling strength of stiffened plates with respect to the width-thickness ratio parameter,  $R_R$ . Here,  $R_R$  is defined as

$$R_R = \frac{b}{t} \sqrt{\frac{\sigma_y}{E} \frac{12(1-\nu^2)}{\pi^2 k_R}} \quad (1)$$

Here,  $b$  and  $t$  are the overall width and thickness of the stiffened plate respectively;  $E$  is the modulus of elasticity and  $\nu$  is the Poisson's ratio of material;  $k_R (=4n^2)$  is the buckling coefficient depends on the number of subpanels,  $n$  divided by stiffeners.

This ultimate strength curve is valid if only the minimum relative stiffness ratio of longitudinal stiffener,

$$\gamma_l / \gamma_{l.req} \geq 1 \quad (2)$$

Here,  $\gamma_l$  and  $\gamma_{l.req}$  is the relative stiffness ratio and required relative stiffness ratio of longitudinal stiffener as per Japanese Specifications of Highway Bridges (JSHB). Except this minimum requirement, the current

strength curve does not represent the capacity of stiffened plates with large flexural rigidity of longitudinal stiffener. Moreover, the strength curve is on the basis of plate-like behavior.

Eurocode differentiate stiffened plates in terms of buckling behavior; plate-like buckling and column-like buckling. Stiffened plates of longer width exhibit column-like buckling are not involved in catenary action results very low curvature in transverse direction and provides no support along longitudinal edges as shown in Figure 2. For this reason, wide stiffened plates of low aspect ratio ( $\alpha = a/b \leq 1.0$ ) regarded as a series of isolated column model whereas column comprises by gross cross section of one longitudinal stiffener associated with a part of the main bottom plate. Here,  $a$  denotes the length of the plate, i.e., the distance between two transverse stiffeners and  $b$  is the overall width of the plate. In this case, increasing stiffener stiffness may significantly affect the strength curve for column-like buckling model.

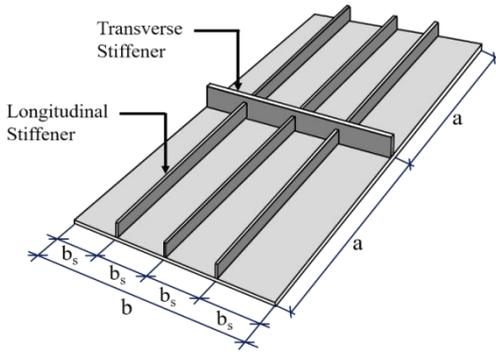


Figure 1. Wide stiffened steel plates.

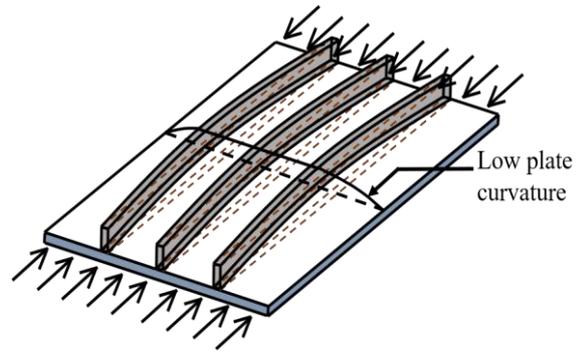


Figure 2. Stiffened plate with column-like behaviour in uniaxial compression.

Nara also addressed this matter by presenting a method to calculate ultimate strength of stiffened plates calls multi-stiffener model considering stiffener geometry which he validated with numerous experiment data. However, Nara didn't differentiate plate-like and column-like buckling in his proposed strength curve of multi stiffener stiffened plate model.

Based on the preceding discussion, this study investigates the effect of flexural rigidity of longitudinal stiffener on ultimate buckling strength focusing column-like buckling stiffened plates model.

## 2 NUMERICAL MODELLING

### 2.1 Wide Stiffened Plate Model Geometry

To achieve column-like buckling in numerical analysis, aspect ratio  $\alpha = 1.0$  with three equidistant open ribs longitudinal stiffeners has been chosen. As wide stiffened plates regarded as a series of isolated column due to unsupported longitudinal edges and also loading and boundary conditions are symmetric, a column model as shaded region in Figure 3 is considered for numerical analysis.

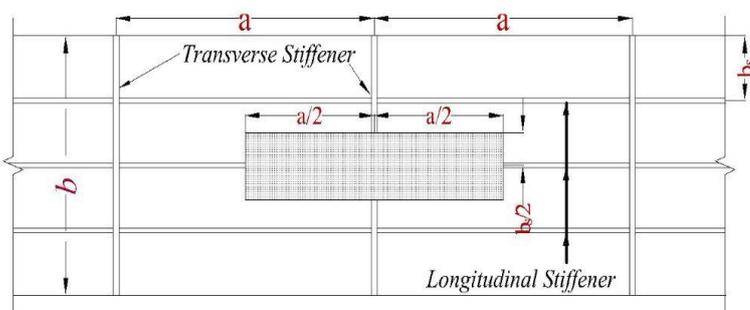


Figure 3. Stiffened plate model.

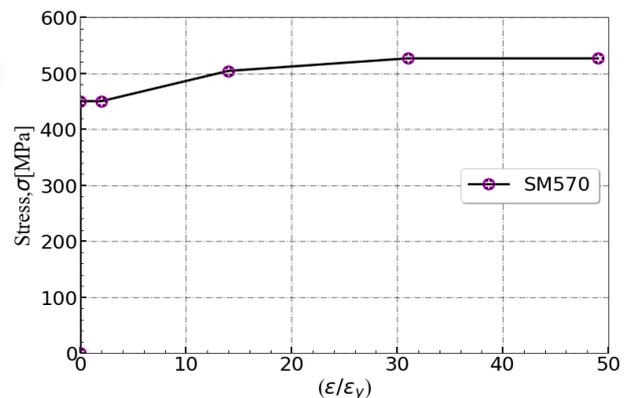


Figure 4. Idealized stress strain relationship for steel grade.

## 2.2 Material Modeling

Steel grades SM570 has been used in present study. The standard value of modulus of elasticity,  $E$  and Poisson's ratio,  $\nu$  are considered as 200 GPa and 0.3 respectively. An idealized uniaxial stress-strain relationship, as shown in Figure 4, based on test data is used to characterize the inelastic behavior of material grade.

Steel grade SM570 is characterized by a yield plateau between start of yielding and the start of moderate uniform strain-hardening. The above stress strain relationship indicates significant ductile behavior of steel grade SM570. Von Mises plasticity, corresponding flow rule, and the isotropic work hardening theory are used to simulate material nonlinearity in numerical analysis.

## 2.3 Boundary Conditions

A complete boundary conditions of column-like buckling wide stiffened plate model is illustrated in Figure 5.

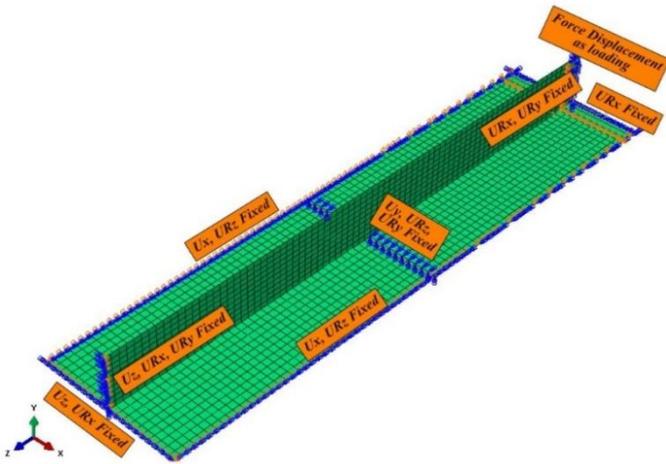


Figure 5. Boundary conditions of wide stiffened column model.

In above figure  $U_x$ ,  $U_y$  and  $U_z$  are denoted translation along the X, Y and Z axes respectively, and  $U_{rx}$ ,  $U_{ry}$  and  $U_{rz}$  are implied as the rotational degrees of freedom with respect to X, Y and Z axes respectively. Column-like buckling behavior can simply be achieved by removing support along longitudinal edges. In the location of transverse stiffener, appropriate boundary conditions comprise equivalent rigidity is applied along the nodal line to make the model computationally efficient. To make sure of uniform compressive stress application to the stiffened plate and stiffener, force displacement is applied as compressive loading.

## 2.4 Initial Imperfections Simulation

In real structural members, different types of imperfections arise, affected their behavior especially steel profiles subject to axial-compression. In present study, 3 types of initial imperfections i.e. residual stress, initial whole plate and local plate out-of-plane deflection has been considered as independent random variables.

Residual stresses are distributed along the width of bottom plate and stiffener. The numerical value of residual stresses can be applied element wise to the model through the "predefined fields" command in Abaqus FEA as initial conditions of "static general" analysis method. The distribution pattern of residual stresses in stiffened plates follows self-internal equilibrium state between the residual tensile and residual compressive stresses ( $\sum \sigma_{rc} = \sum \sigma_{rt}$ ) has shown in Figure 6 below.

Here,  $\sigma_{rc}$  and  $\sigma_{rt}$  are the residual compressive stress and residual tensile stress respectively, and  $\sigma_y$  is the yield stress of the respective plate.

The simulation of initial out-of-plane deflections is applied in Abaqus FEA by the method of linear superposition of desired buckling mode shapes. At first, relevant buckling mode shapes is determined by visual inspection from mode shapes of elastic buckling analysis and then scaled to the desired magnitude of out-of-plane deflection and incorporate to the perfect geometry of stiffened steel plates model. The simulation of whole plate and local plate out-of-plane deflections to the desired statistical value has shown in Figure 7 as an example.

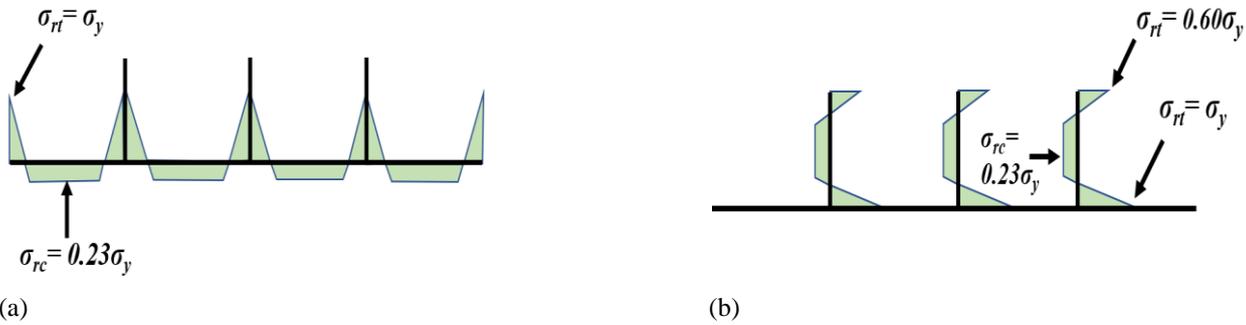


Figure 6. Residual stress distribution pattern for (a) bottom flange and (b) longitudinal stiffener.

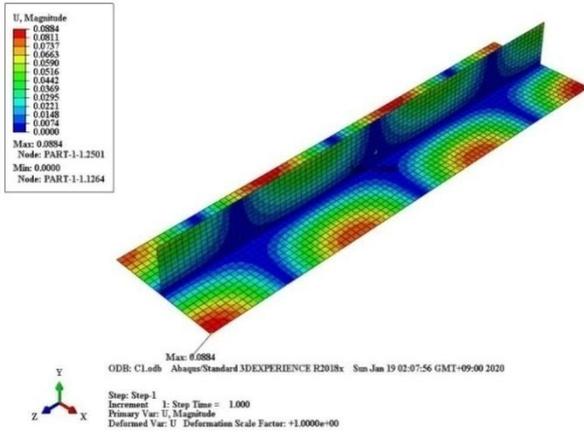


Figure 7. Simulation of two relevant modes to the stiffened plate model of  $R_R$  0.8 and  $\gamma_l/\gamma_{l.req} = 3$ .

### 3 FINITE ELEMENT ANALYSIS & RESULTS

Commercial code *Abaqus FEA* is used to simulate stiffened plates with stiffener with the four-node deformable, reduced integration, hourglass controlled shell elements designated as S4R in finite element analysis.

#### 3.1 Elastic Buckling Analysis

The whole plate and local plate out-of-plane deflected mode shape; reflecting the field conditions of wide stiffened steel plates after fabrication and erection, were visually analyzed from several buckled shapes calculated by elastic buckling analysis. Linear elastic critical buckling strength,  $\sigma_e$  has taken as the lowest critical stress of relevant buckling modes. The influence of flexural rigidity of longitudinal stiffener to the elastic buckling strength calculated from numerical analysis for  $R_R$  1.0 has shown in Figure 8 as an example. It was found that, local buckling of sub-panels dominates for large flexural rigidity of stiffeners irrespective of  $R_R$  value.

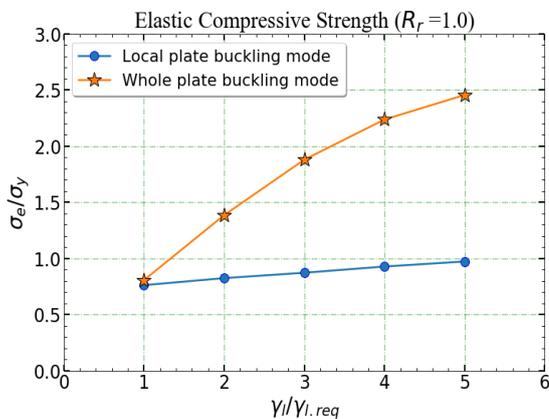


Figure 8. Effect of relative stiffness ratio of stiffener on elastic critical strength.

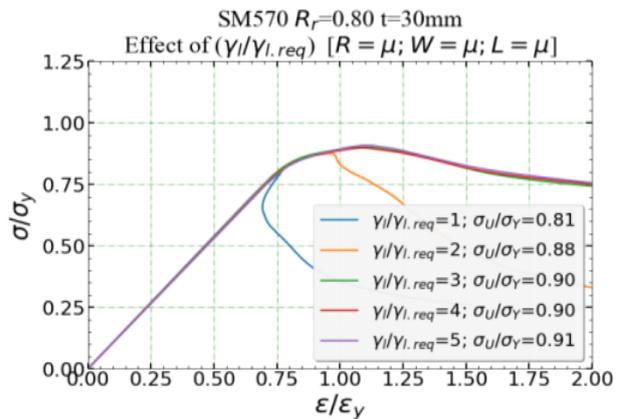


Figure 9. Effect of variation of relative stiffness ratio on ultimate buckling strength for  $R_R$  0.8

### 3.2 Nonlinear Elasto-Plastic Analysis

Nonlinearity of material properties and geometric configurations has been considered to obtain ultimate capacity of longitudinally wide stiffened steel plates. The effect of the uncertainties of initial imperfections is accounted by considering 7 combinations among the imperfections based on reported statistical data (Fukumoto et al. 1984, Nara et al. 1988, Komatsu et al. 1980) as shown in Table 1.

Table 1. Statistical parameter of initial imperfections.

Imperfections	Mean value ( $\mu$ )	Standard deviation ( $\sigma$ )
$\frac{\sigma_{rc}}{\sigma_y}$	0.230	0.145
$\frac{1000\delta_{01}}{a}$	0.096	0.426
$\frac{150 \Delta_{ini} }{b_s}$	0.138	0.107

Here, initial whole plate and local plate out-of-plane deflections are symbolized by  $\delta_{01}$  and  $\Delta_{ini}$  respectively and  $b_s$  is the width of a sub-panel between two longitudinal stiffeners. Total 210 stiffened steel plates model has been analyzed by nonlinear elasto-plastic analysis. With increasing the relative stiffness ratio of longitudinal stiffener, ultimate strength is found increased as anticipated. Figure 9 depicts the effect of relative stiffness ratio of longitudinal stiffener on Ultimate Limit States (ULS) of  $R_R$  0.8 for all mean value imperfections as an example.

## 4 PROBABILISTIC BUCKLING STRENGTH BY APPROXIMATE SOLUTION

The inconsistency of ultimate buckling strengths of stiffened plates is addressed by three random variables. Mean and variances of ULS estimated from unknown relationship between the multiple random variables by approximate solution.

In approximate solution procedure (Halder et al. 1999), the ultimate buckling strengths expressed in terms of random variables as follows:

$$Y = g(R, W, L) \quad (3)$$

where,

$$Y = \frac{\sigma_{ULS}}{\sigma_y};$$

Three initial imperfections denoted by,

$$R = \frac{\sigma_{rc}}{\sigma_y}; \quad W = \frac{1000\delta_{01}}{a}; \quad L = \frac{150\Delta_{ini}}{b_s};$$

where,  $g$  is the response function of random variables. Random variables  $R$ ,  $W$  and  $L$  are normalized residual stress, initial whole plate and local plate out-of-plane deflections respectively. First order approximate mean value of ultimate buckling strengths is estimated by analyzing stiffened plates with a combination of all mean value initial imperfections. After that, Taylor series finite difference (TSFD) estimation procedure employed to estimate the variance of ultimate buckling strengths numerically. Then, each stiffened plate model analyzed two times more corresponding to each random variable i.e. mean plus one standard deviation ( $\mu+\sigma$ ) and mean minus one standard deviation ( $\mu-\sigma$ ). Partial derivatives of ultimate capacity with respect to random variables estimated through the central difference approximation and finally first order variance of ultimate strengths obtained by using partial derivatives corresponding all random variables as follows:

$$Var(Y) \approx \left(\frac{Y_R^+ - Y_R^-}{2\sigma_R}\right)^2 + \left(\frac{Y_W^+ - Y_W^-}{2\sigma_W}\right)^2 + \left(\frac{Y_L^+ - Y_L^-}{2\sigma_L}\right)^2 \quad (4)$$

## 5 RESULTS & DISCUSSIONS

The probabilistic ULS strength at  $R_R$  1.2 with respect to relative stiffness ratio of longitudinal stiffener has shown in Figure 10.

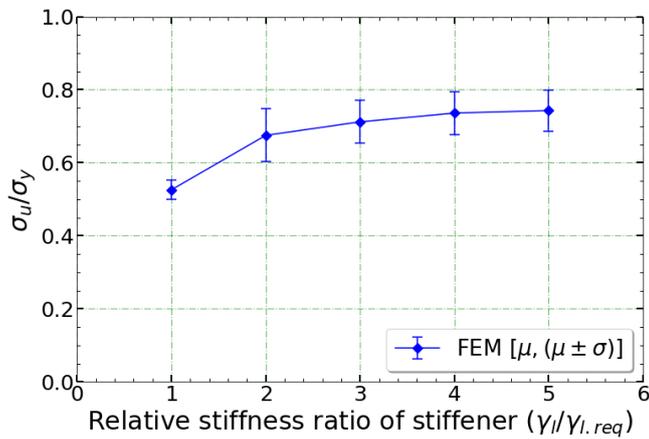


Figure 10. Variation of probabilistic strength with respect to relative stiffness ratio of stiffener at RR 1.2

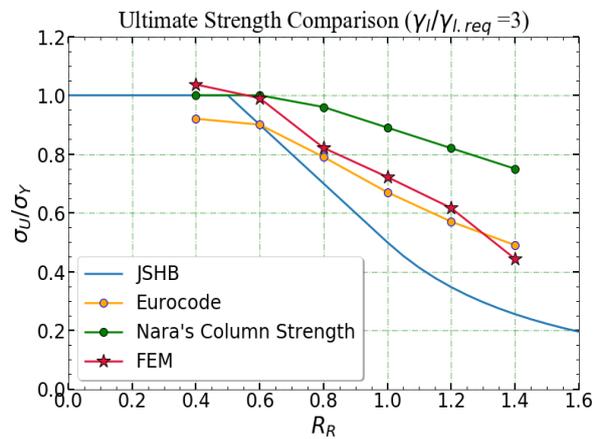


Figure 11. Comparison of ULS obtained by approximate solution with other design codes and proposal.

In this figure, midpoints represent the mean value ( $\mu$ ) strengths and top and bottom of error bars represent the strengths corresponding to  $(\mu + \sigma)$  and  $(\mu - \sigma)$ , respectively. It was observed that the ultimate strengths increased more significantly up to relative stiffness ratio of 3. The above plot also implies that, inconsistency of compressive strength is increased for large flexural rigidity of stiffener.

Finally, lower limit (5% fractile) of probabilistic strengths calculated assuming normal distribution of compressive strengths and rationally compared with design codes e.g. JSHB, Eurocode and Nara's proposed column strengths. Figure 11 shows such results for relative stiffness ratio of longitudinal stiffener value of 3.

It was found that, Eurocode strength prediction matched well with Finite Element Method (FEM) results while, JSHB underestimates and Nara overestimates the strengths of wide stiffened plates greatly at  $R_R \geq 0.8$ . The results of the present study can be referenced to develop a design guideline for ultimate strength curve of stiffened steel plates exhibiting column buckling behavior.

## REFERENCES

- Beg, D., Kuhlmann, U., Davaine, L., & Braun, B. 2010. *Design of Plated Structures: Eurocode 3: Design of Steel Structures, Part 1-5 - Design of Plated Structures*: First Edition.
- Fukumoto, Y. & Itoh, Y. 1984. *Basic compressive strength of steel plates from test data*, JSCE Struct. Eng. Earthquake Eng., 344 (I-1) 129-139.
- Haldar, A. & Mahadevan, S. 1999. *Probability, Reliability, and Statistical Methods in Engineering Design*. First ed., John Wiley and Sons, New York.
- JSHB, 2002. *Specifications for Highway Bridges: Part-II Steel bridges*, Japan Road Association.
- Komatsu, S., Kitada, T., Watanabe, E., Akiraumi, M. & Nara, K. 1980. *Statistical study of the initial deformation and the ultimate strengths of steel bridge members*, JSSC, 16, 10-43. (In Japanese)
- Nara, S. 1992. *Evaluation of ultimate strength of longitudinally stiffened continuous plates under uniaxial compression*, Proc. of JSCE, 38A, 289-296. (In Japanese)
- Nara, S. & Komatsu, S. 1988. *Statistical study on ultimate strength curves of longitudinally stiffened plates under uniaxial compression*, JSCE Struct. Eng. Earthquake Eng., 392 (I-9) 289-296. (In Japanese)
- Ziemian, R.D. 2010. *Guide to stability design criteria for metal structures*, sixth ed., John Wiley & Sons Inc., New Jersey.